Institute of Acoustics

DIPLOMA IN ACOUSTICS
AND NOISE CONTROL

A Study of French Horn Harmonics

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Q: How do you make a trombone sound like a French horn?

A: Put your hand in the bell and play all the wrong notes.

ABSTRACT

- The acoustic theory of sound production and the harmonic series of the French horn are examined.
- A summary of how brass instruments function acoustically is presented and related to the theory of resonance in closed-open cylindrical pipes.
- The harmonic series produced by a horn is measured and analysed with an FFT analyser.
- The horn is modified with various lengths of cylindrical hosepipe to study the effect of the conical mouthpiece and mouthpipe and the flaring bell separately.
- The experiments indicate that the mouthpiece, mouthpipe and bell modify the tuning of harmonics of the simple cylindrical pipe in a frequency-dependent manner.
- The lowest note playable on the horn is found not to be a natural resonance of the instrument, but a heterodyne tone produced by interaction between the upper harmonics. This note is called the ‘pedal note’.
- The mouthpiece and mouthpipe have a relatively small effect on the tuning. The bell significantly changes the tuning of the lower harmonics so that the horn appears to play a full set of odd and even harmonics from the pedal note upwards.

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CONTENTS

1 INTRODUCTION

There cannot be many brass players who have not tried to play a piece of hosepipe for fun at some point in their careers. The procedure is simple – place the instrument's mouthpiece in the end of the hose, a kitchen funnel at the other end, and literally minutes of amusement are sure to follow.

An interested party might notice that the hosepipe instrument plays a slightly different set of pitches to a real instrument, and that many notes are out of tune. On the surface there would seem to be no logical reason why the horn should be any different to any bit of central heating pipe or hosepipe. The purpose of this project is to study the acoustic reasons why these differences occur.

1.1 Scope

The basic objectives of this project are as follows:

- To undertake a study of the basic physical mechanisms enabling the horn to produce sound.
- To predict the harmonic series of a simple closed-open cylindrical pipe.
- To take measurements of the natural resonances of the simple pipe and horn.
- To analyse the results to explain any differences.

1.2 Structure of this project

- Section 2 sets out a brief discussion of the instrument to provide some background to the study. This contains a short history of its development, a description of the modern instrument and the method of playing, and some discussion of its uses.
- Section 3 describes the physics of sound production and the role of each part of the instrument.
- Section 4 sets out acoustic measurements to study the harmonic series.
- Section 5 presents an analysis of the experimental evidence versus the theory.
- Section 6 sets out a summary of conclusions.
- Appendix A sets out references and a bibliography.

- Appendix B describes an interview with the trumpet maker Andy Taylor, and some notes on how practical experience of instrument design and manufacture relate to the studied theory.
- Appendix C describes the measuring equipment for the experiments.
- As much of the project concerns the subjective sensation of pitch, it is much easier to hear the effects described rather than interpret them from graphs and data. An audio CD is therefore provided with the project with illustrative audio examples, and Appendix D contains a track listing for the CD.
- Although this project is mainly concerned with the physics of pipes, the reader will need to have a very basic grasp of the musical theory of notes and pitches. To help with this, Appendix E contains a glossary of some of the terms used in this project.

1.3 A note on nomenclature

Before the main discussion begins, it is useful to clarify some terminology used to describe notes produced by pipes and musical instruments. The theory of resonance in pipes is discussed fully later in this project, but as the terminology is used throughout this project in both physical and musical contexts it is helpful to establish this before going any further.

- A pipe has a series of natural resonances called *modes*. The lowest-frequency mode is known as the *fundamental.*
- In mathematical terminology, resonances that are integer multiples of the fundamental are called *harmonics*. The natural resonances produced by brass instruments are generally not exact multiples of the fundamental, but are still referred to as harmonics for convenience. The harmonics are numbered from the fundamental upwards, so the fundamental is the first harmonic, the next resonance the second harmonic, and so on.
- Another term for describing the mode number is the *overtone*. These commence their numbering from the resonance following the fundamental, so the first overtone is the second harmonic.
- An instrument can oscillate in several modes simultaneously, and each of these modes are referred to as *partials* or ‘partial vibrations’.

- Where the names of notes are referred to, these are in the so-called ‘scientific pitch notation’, where notes are categorised by letter name, accidental (sharp or flat), and octave number.

2 THE FRENCH HORN

2.1 Origins and development

2.1.1 The hunting horn

The modern-day horn is a direct descendent of the animal horn used as a signalling device by ancient man. The narrow end of a hollow animal horn would have been cut off to provide an opening to blow into the instrument. The sound would have been loud and rough and capable of only one or two notes, so although effective as a signalling device would have had limited scope for use as a musical instrument.

Figure 1 - "How to shout and blow Horns." Facsimile of a Miniature in the Manuscript of Phoebus (Fifteenth Century)

2.1.2 Musical use – the natural horn

It is estimated that horn-makers began to experiment with the instrument for musical use in the 16th century [Tuckwell, 1983, p. 10]. In the field, the English had been using a shorter horn with a more bugle-like tone, whereas the French favoured a looped instrument consisting of a long conical section with a short terminal flare, approximately 2 metres in length [Campbell / Greated, p. 392]. These instruments would have been capable of a restricted range of notes, and to increase this range a long central cylindrical section was introduced to the French instrument to bring the overall length to around 4 metres. Further development of the instrument meant that, by the end of the 17th century, contemporary accounts indicate that skilled players were able to play as great a range of notes as the trumpeters of the day [Tuckwell, 1983, p. 11]. Track 1 on the CD demonstrates this versatility in an excerpt from Handel's *Water Music*, where the horn players can be heard to play rapid passages and trills. *Water Music* is commonly said to be the first piece of music played in England using horns as musical instruments.

In the late 17th Century, the Leichnambschneider brothers in Vienna developed an instrument they called the 'Waldhorn' with a larger bore and wider throated bell, which produced a mellower and less strident tone than the French hunting horn. This marked the evolution of the horn into an instrument resembling the modern version. Figure 2 shows a horn from this period by the French manufacturer Raoux.

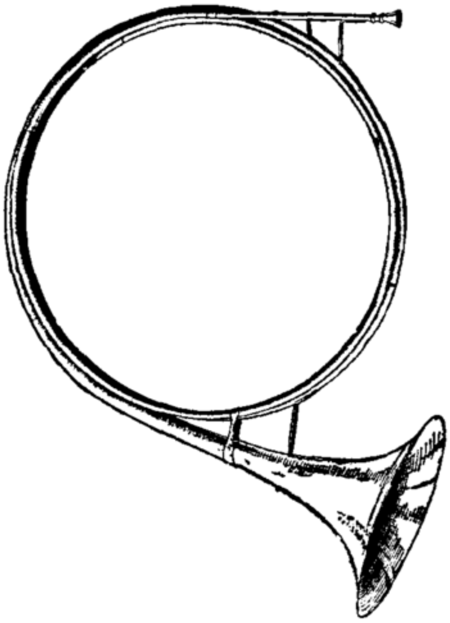


Figure 2 - Early Raoux Horn

These early instruments would have only been able to play a limited number of notes in the natural harmonic series of the pipe, which would have restricted its use to compositions in a single key. Furthermore, some of these notes would have been out of tune with the normal musical scale. Figure 3 shows the notes of the harmonic series of a horn with a fundamental pitch of C, and the out-of-tune notes are highlighted in blue.

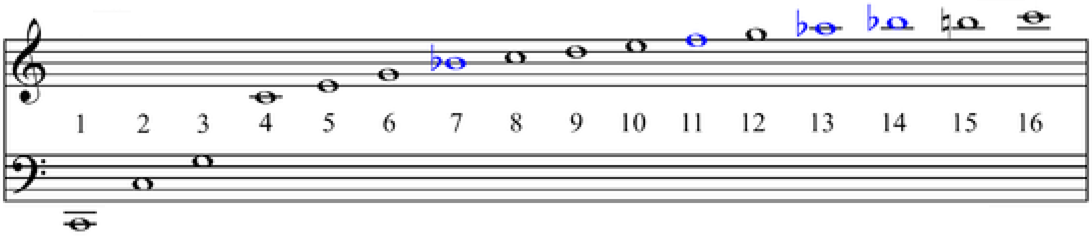


Figure 3 - Natural harmonics of horn with basic pitch of C

This limited range of pitches meant that early horn music is often written mostly in the upper range of the instrument, as basic tunes could be more easily produced from the notes that are close together.

The basic range of pitches is related to the length of the pipe (see Section 3.2). Developments in the 18th Century allowed additional lengths of tubing known as crooks to be inserted into the instrument to lengthen the pipe and change the harmonic series, and hence allow the horn to be played in different keys.

The mid 18th century saw further development of playing practice when the Dresden player Hampel found a method of altering the natural pitches by placing the hand into the bell, called *hand stopping*. By doing this it was found that the pitch of the instrument could be lowered in varying degrees up to a few semitones, and by doing this some missing notes of the scale could be filled in. Hence the instrument, which had up to this point been used with the bell pointing upwards, became used as we know the modern version – pointing downwards and backwards. The hand in the bell also altered the tone of the instrument, producing an attractive darker, more veiled quality. The repertoire of the instrument could therefore be greatly extended to take advantage of the increased available notes, and composers began to write more interesting orchestral and solo works.

2.1.3 The chromatic instrument

A drawback of the hand-stopped natural horn was that although more notes were available than on the basic instrument, the entire chromatic scale could not be played. Furthermore, the tone of the stopped notes was more muffled than the open notes, and a consistent tone quality was not possible over the instrument's range. This can be heard in an excerpt from Mozart's *Second Horn Concerto* on track 2 of the CD, where some of the notes can clearly be heard to be stopped (for example, at time 5:03).

Manufacturers therefore experimented with technical systems that would allow the player to alter the pitch of the open notes by adding short extra lengths of pipe to the basic instrument, rather than using the hand. An interesting example of a horn from this period is the 'Omnitonic' horn invented by DuPont in Paris around 1815. This instrument had eight separate tubes of different lengths combined into one instrument, and the mouthpiece was placed into the pipe corresponding to the required key. Though undoubtedly an extraordinary piece of engineering, this instrument must have been rather confusing to play – not to mention heavy!

Figure 4 - Omnitonic horn

Any further development of this type was stalled by the invention of the air valve in the early 19th Century. Valves enabled the player to add extra lengths of tube to the instrument at the press of a key. By carefully choosing the lengths of pipe the open notes could be lowered by defined intervals, enabling a full chromatic range of notes to be played by 'filling in' the open notes.

The basic principle of a valve is shown in Figure 5. When the valve is open (as shown on the left) the air flows uninterrupted through a straight piece of pipe. When the valve is closed the air is diverted through an extra piece of pipe which makes the overall instrument longer, and hence lowers the pitch by an amount related to the length of the extra pipe.

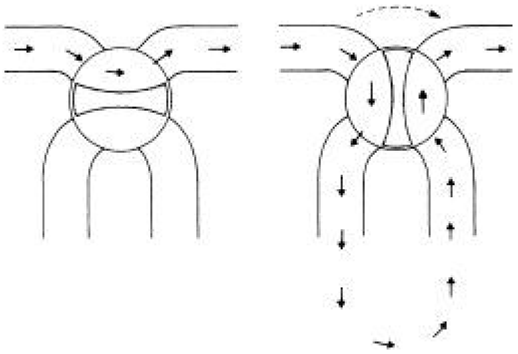


Figure 5 – Rotary valve open (left) and closed (right)

Composers were quick to adopt the increased musical opportunities from the new chromatic instrument into their music, and thus the modern instrument as we know it was born. The modern horn has in fact changed very little from those built in the 1820s [Tuckwell, 1983, p 57]. Track 3 on the CD is an excerpt from the *Second Horn Concerto* by Richard Strauss, which is extremely chromatic, and would have been impossible to play on the natural horn.

2.2 The modern instrument

The most commonly used modern instrument is the so-called *double horn*, which uses two primary lengths of pipe, and hence two basic sets of harmonic series, in a single instrument. The two keys selected are F and B♭, and the reason for this is that the playing characteristics are slightly different between the two instruments (see Section 2.4). The two 'sides' of the horn are selectable by a thumb-operated valve, and three other valves add in the short additional lengths of pipe called *tuning slides* to enable a full chromatic scale to be played. The tuning slides can be moved in and out to make fine adjustments to the tuning and intonation, and they can also be removed to enable the natural built-up of water in the instrument to be emptied out (this is, contrary to the disgust of other instrumentalists, not saliva but mostly condensation!)

The main sections of the horn are labelled in Figure 6:

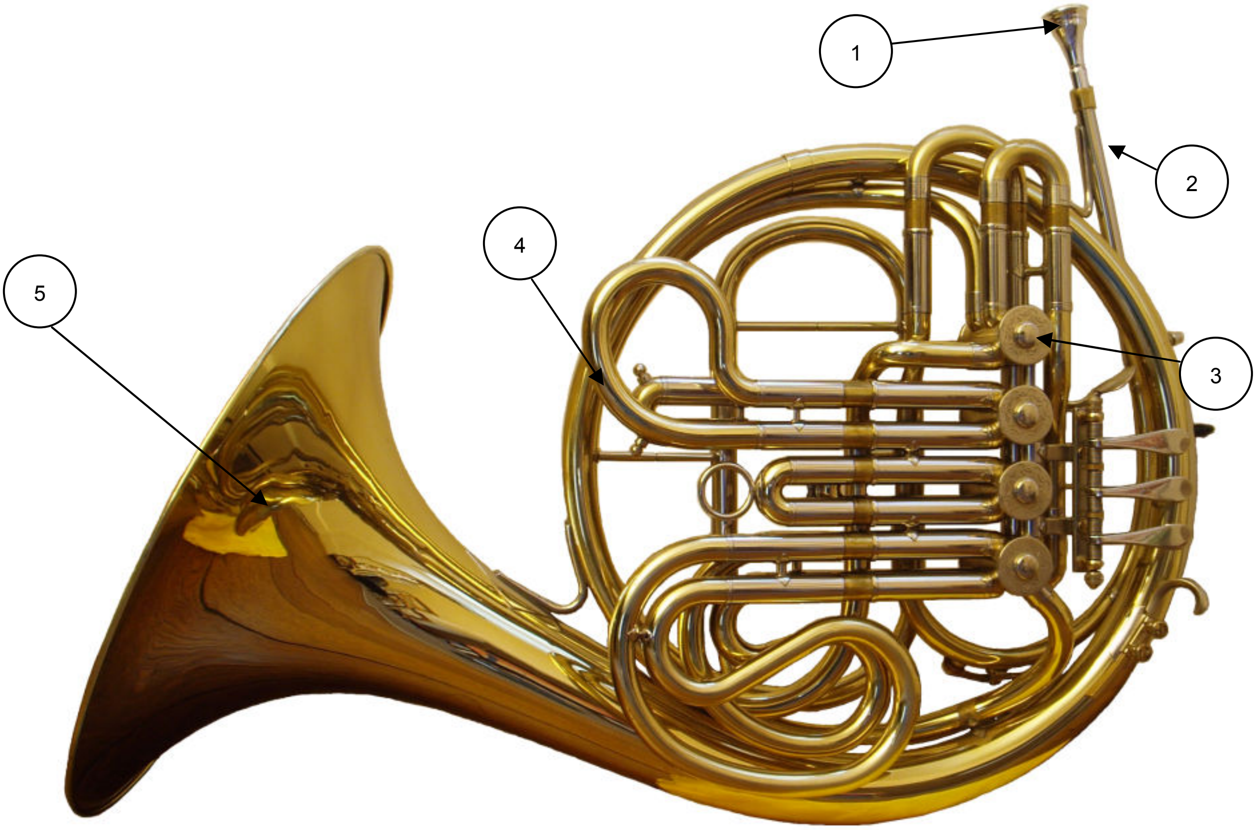


Figure 6 - Main horn components

1. **Mouthpiece**: The instrument starts with the mouthpiece, which is pressed to the lips to form an airtight seal.
2. **Mouthpipe:** The first section of pipework following the mouthpiece, which is roughly conical in profile. The length of this section varies between instruments, but is typically only 1/7 of the length of the horn [de Haro horns website]. The pipework from the end of the mouthpipe to the start of the bell flare is cylindrical (including the tuning slide section).

3. **Valves:** Rotary valves add in the tuning slides to lower the pitch of the open notes. The first valve/tuning slide lowers the open notes by a tone, the second a semitone, and the third one and a half tones. The fourth valve (operated by the thumb) sets the basic length of the pipe to the F or B♭ harmonic series.

4 **Tuning slides:** The tuning slides lower the pitch of the open notes when switched in by the valves as described in (3). The slides can be moved in and out to fine-tune individual notes and can be taken out completely to empty water accumulated during playing.

5 **Bell:** The instrument terminates with the bell, from which all of the sound radiates. The French horn bell is much larger than that of a trombone, but smaller than that of a tuba.

2.3 Method of playing

The instrument is held with the left hand, supported by a finger hook:

Figure 7 - How to hold the horn

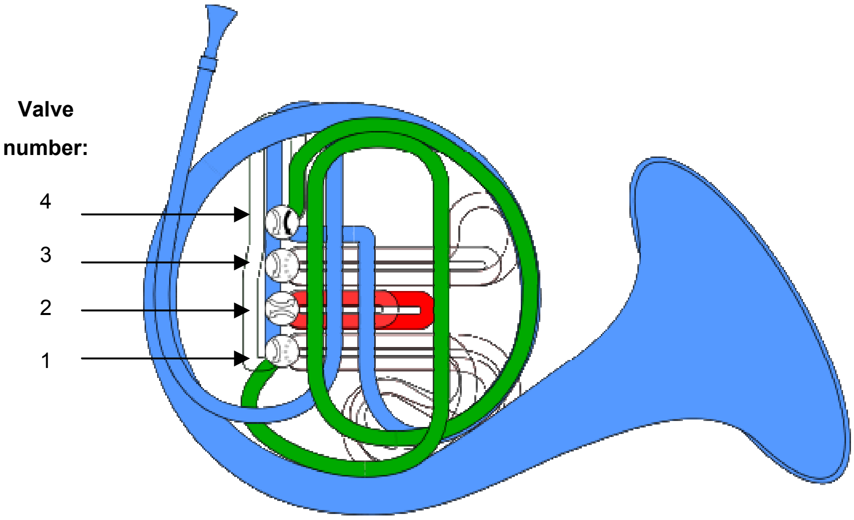
The bell is pointed to the rear and the right hand is placed in the bell, not completely blocking the airflow, but held as shown in Figure 8.

Figure 8 - Hand in the bell

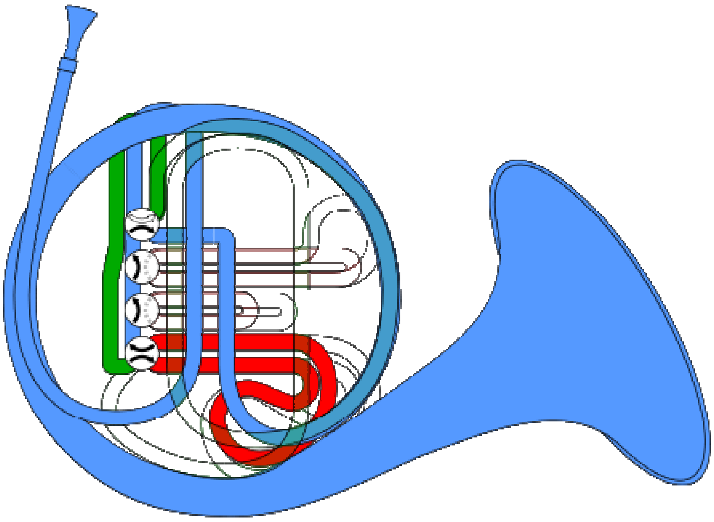
Notes cannot be produced from the horn by simply blowing down the end of the instrument - doing this will just quietly expel air out of the other end. The player produces a note by placing the lips on the mouthpiece to form an effectively airtight seal, holding the lips in tension, and blowing through them to cause a vibration. It is often said that the player blows a raspberry down the instrument, but the buzzing is actually very finely controlled. The act of puckering the lips is known as ‘forming an embouchure’.

At certain lip tensions the instrument will spontaneously ‘speak’ and a note will be produced. The frequency (pitch) of the note will be one from the harmonic series dictated by the length of the pipe, termed the ‘open’ notes. Varying the lip tension enables the note to be changed to another in the harmonic series.

The valves are used to provide a greater range of notes by lowering the pitch of the open notes by various degrees. The first three valves are designed to lower the open notes by a tone, semitone, and three semitones respectively. The fourth valve, operated by the thumb, switches the main harmonic series between the F and B♭ sides of the instrument. Figure 9 shows the valves in use:



Case 1: The fourth valve (top) is open, selecting the F side of the horn (long pipe, shown in green). The second valve (third from top) is depressed to lower the open notes by a semitone by selecting the appropriate tuning slide (shown in red).



Case 2: The fourth valve is depressed, selecting the higher pitched B♭ side (shown in green). The third valve is depressed to lower the open notes by three semitones (shown in red).

Figure 9 - Use of the valves

2.4 Notes produced by the instrument

As discussed in Section 2.1.2, the basic pitch of the harmonic series is dependent on the length of the instrument. The double horn effectively contains two lengths of pipe of approximate basic length 12 and 9 feet selectable by valves. This means that two fundamental harmonic series are selectable commencing with the notes F and B♭ respectively. One of the reasons for this is that in the higher ranges of the instrument the harmonics of the F horn are very close together. It is generally easier to 'hit' a given note when the harmonics are further apart, as less fine gradations in lip tension are required to change notes. As the B♭ horn is pitched higher than the F horn, the harmonics are further apart, and the B♭ horn is therefore often used more in the higher register.

The reason that all horns are not pitched in B♭ is that the tone quality is different between the F and B♭ sides, and the player may therefore wish to use different sides in certain ranges to provide a particular sound. However, many players (the author included) play throughout most of the range on the B♭ horn by preference.

2.5 Repertoire

The repertoire of the horn is varied, and the instrument is used extensively in both orchestral and solo work. Following Hampel's development of the hand horn technique, baroque and classical horn parts can be categorised as either very simple or advanced, with very little between the extremes. For example, complex and difficult horn parts can be heard in the Bach Brandenburg Concerti, which would have been played on natural horns relying on hand stopping to produce notes not in the natural harmonic series. However, much of the symphonic writing by Haydn and Mozart is very straightforward for the horn player, being based around the natural notes of the harmonic series.

Some of the most famous music written for the horn is the four horn concertos written by Mozart. The pieces would have been written for the natural horn, and there are many passages that would have required great flexibility from the player. On modern instruments the concertos are easier with the use of valves, but many passages still have difficult sections.

The horn began to feature in the orchestra with increasing prominence in the Romantic period, with composers writing more adventurous parts for the new chromatic instrument. Early romantic composers such as Schumann wrote extensively for the horn in their symphonies, as well as several solo works - notably the Konzertstück for four horns, which even for modern players is very difficult.

Romantic composers such as Wagner featured the horn extensively in his large-scale operas. Richard Strauss wrote important horn parts in his music, and as well as writing two difficult concerti which are standards of the repertoire, wrote extensive and prominent horn parts in his tone poems. Compositions such as the *Alpine Symphony* feature up to ten horns in a huge orchestra.

One of the unique features of the horn is the ability to produce a range of tone qualities. The horn is capable of a brassy sound to match the other brass instruments, but can also blend well with the woodwind and strings in orchestral accompaniments.

Track 4 on the CD is an excerpt from Strauss's *Also Sprach Zarathustra*, demonstrating the brassy sound quality of the instrument, which is a feature of much of the Romantic horn repertoire.

Whereas the trumpets generally have too bright a tone to accompany other instruments, the horn has a softer sound well-suited for this purpose. Track 5 on the CD is an excerpt from Sibelius's *Violin Concerto*, where only the horns and bassoons provide the main sustained accompaniment.

2.6 Quirks of the instrument

The French horn has a reputation for being a difficult instrument to play. It is not unusual when attending orchestral concerts to hear even the finest horn players 'crack' a few notes, or in the case of less fine horn players, occasionally blow them to pieces. This is generally accepted by other orchestral players, used to hearing such blemishes, but to the unaccustomed audience member it often comes as a surprise to hear professional players 'fall off' notes.

Barry Tuckwell in the foreword to his book *Horn* states that this reputation is not quite true, as all instruments have their difficulties, but that playing the horn is more *treacherous* than most instruments. The reason for this is to mostly to do with the close spacing of the harmonics in the normal playable range as described in Section 2.4. Other brass instruments such as the trumpet have harmonics much further apart in the playable range, and there is therefore much more 'margin for error' than while playing the horn. As well as 'hitting' notes in isolation, moving between notes can also be difficult for similar reasons, particularly in fast passages or long leaps between high and low notes.

The higher notes in particular are much more difficult to pitch than the lower notes. These are even more difficult when played quietly. An orchestral horn player's worst nightmare is to be required to play a quiet, high, exposed entry.

The horn is a relatively fatiguing instrument to play. This is probably because the instrument has a relatively narrow bore (tube diameter), and is long in length – approximately 12 feet for the F horn. The trumpet by comparison is around 4 feet in length and has a wider bore, providing less resistance to the player. The horn is unique in the orchestra in sometimes having a 'bumper' player for the first horn who takes over in less important but taxing sections of a work, allowing the principal horn to regain strength.

3 THE PHYSICS OF SOUND PRODUCTION

3.1 Overview

The horn belongs to the family of musical instruments called aerophones - those that produce sound from vibrating columns of air, and where the vibration of the body does not add significantly to the sound. The horn is often described as having a conical bore, i.e. a gradual increase in cross-sectional area from the mouthpiece to the rapidly-flaring bell. This is not strictly correct, as the horn actually has a ‘hybrid bore’ consisting of a conical mouthpiece and mouthpipe, a long cylindrical section (which contains the valves, if fitted), and a long flared section terminating in the bell. This is similar to the trumpet and trombone, but the horn has a much longer mouthpipe and bell flare relative to the central cylindrical section [Campbell/Greated, 1987, p. 392].

The basic mechanism of sound production is the setting up and maintaining of longitudinal standing waves in the pipe. The lips are sufficiently closed to mean that the instrument is effectively a pipe closed at one end and open at the other. A pulse of air is introduced to the end of a pipe from the player via the lips, which produces a pressure wave travelling at the speed of sound. The wave impedance in the pipe is relatively high compared with the atmosphere, and when the wave reaches the end it suddenly meets a low impedance. As a result of this impedance mismatch a proportion of the wave will be reflected and will travel back down the pipe. The non-reflected portion of the wave travels out of the pipe and produces the sound that we hear.

The theory of superposition dictates that at certain frequencies a standing wave will be set up where the incident and reflected waves combine to produce a stationary pattern of vibration. Notes are therefore produced at these resonant frequencies of the pipe.

3.2 Theory – the closed-open cylindrical pipe

A closed-open cylindrical pipe will have natural resonant frequencies related to the length of the pipe, and the boundary conditions dictate the modes of resonance that can be sustained. To set up a standing wave a closed-open pipe must have a pressure antinode at the closed ends and a pressure node at the open end. The pressure distribution of waves in a closed open pipe is shown in Figure 10:

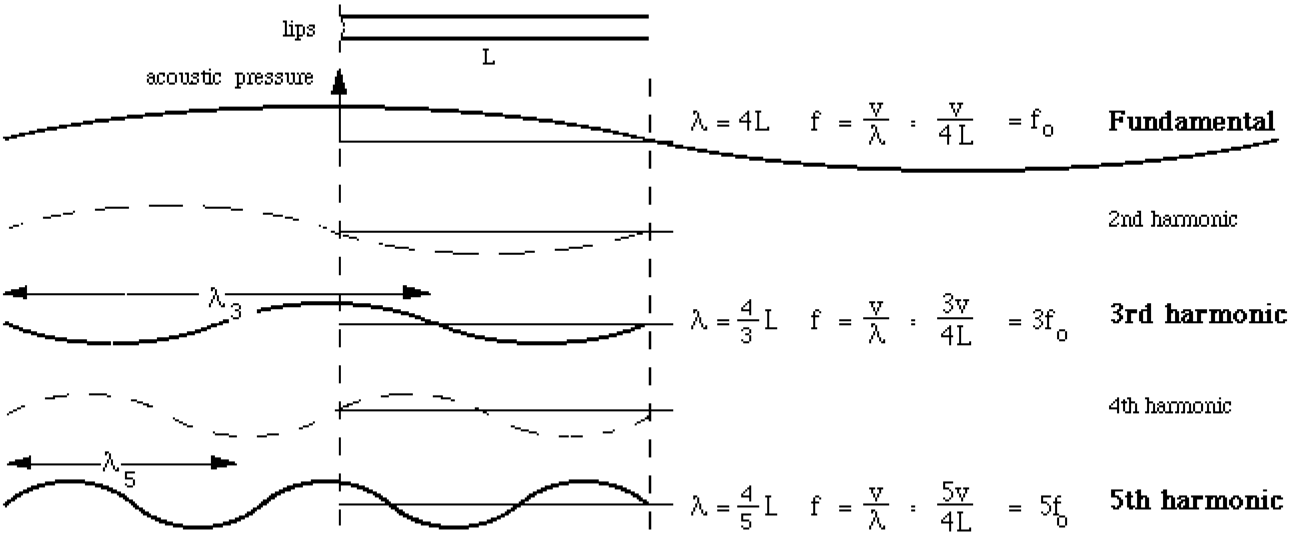


Figure 10 - Pressure distribution and boundary conditions in a closed-open pipe (from http://www.phys.unsw.edu.au/jw/brassacoustics.html)

This means that the lowest frequency of oscillation (fundamental) occurs at a ¼ wavelength, i.e.:

$\lambda = 4L$, and hence $f_0 = \frac{c}{4L}$

where

λ is the wavelength (m)

f_0 is the fundamental mode (Hz)

c is the speed of sound (m/s), and

L is the length of pipe (m)

The boundary conditions are next satisfied where $\lambda = \frac{4L}{3}$ and hence $3f_0$. Further oscillations occur at odd multiples of the fundamental, so a cylindrical closed-open pipe therefore only produces odd harmonics (1, 3, 5, 7, etc.)

3.3 Theory – the closed-open conical pipe

Conical pipes closed at one end are different to cylindrical pipes in that they produce a full set of both odd and even harmonics. This can be explained by examining the way in which sound waves travel down the pipes in terms of the sound intensity.

Sound intensity is defined as sound power over cross sectional area. Sound travels down cylindrical pipes as plane waves, and as the pipe has a constant cross-sectional area the intensity remains the same throughout the pipe (ignoring frictional and other losses).

In contrast, conical pipes gradually increase in cross-sectional area, which means that the wavefront diverges and hence the intensity decreases as the sound wave travels down the pipe. The sound intensity in the pipe is proportional to $\frac{1}{r^2}$, where r is the distance from the apex of the cone, and as pressure squared is proportional to intensity, the sound pressure is therefore proportional to $\frac{1}{r}$.

As discussed in Section 3.2, the boundary conditions of a closed-open pipe mean that there must be a pressure antinode at the closed end and a pressure node at the open end. In the cylindrical pipe these boundary conditions can be satisfied with a cosine wave of wavelength $\frac{4L}{n}$, where L is the length of the instrument and n is an odd integer. In the conical pipe the $\frac{1}{r}$ term must be accounted for, and hence the standing wave has a pressure envelope which is $\frac{1}{r}$ times a sine wave of wavelength $\frac{2L}{n}$, where n is an integer. The function goes to zero at the open end of the pipe where $r = L$, and $\frac{1}{r}\sin r$ has a maximum at the closed end, as required. Figure 11 illustrates this behaviour diagrammatically.

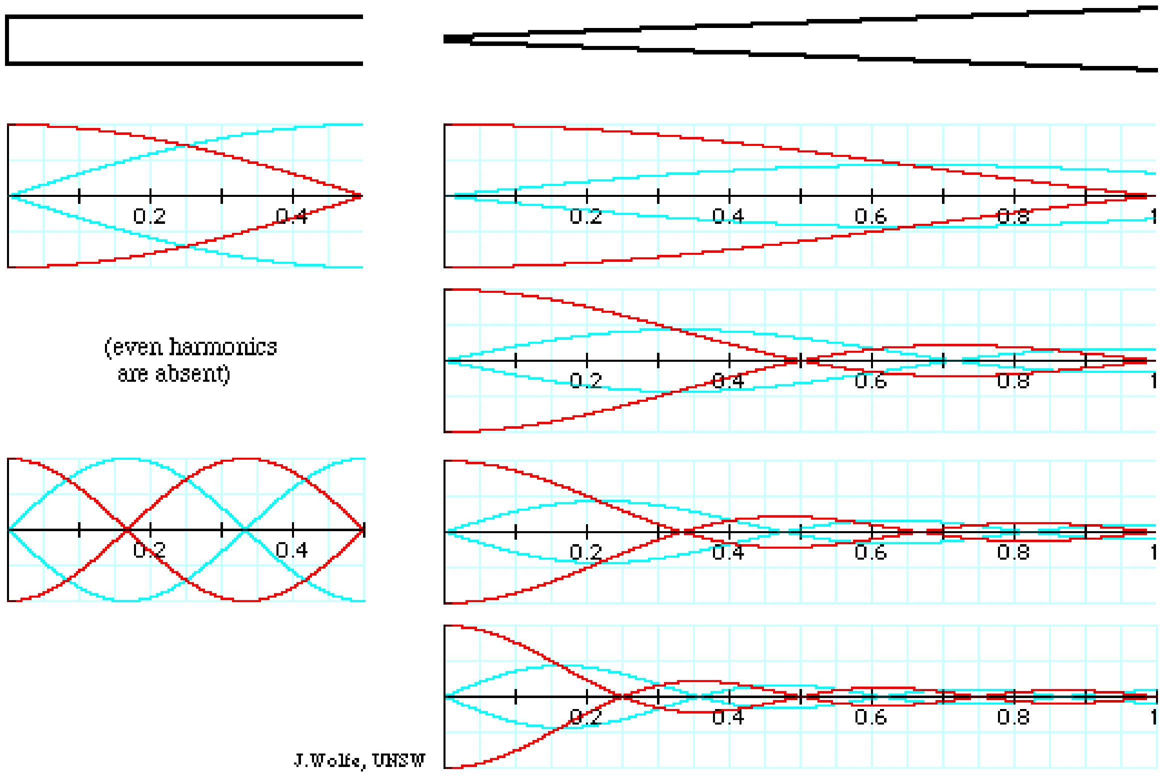


Figure 11 - Pressure and air motion amplitude variations in closed-open cylinder (left) and closed-open cone (right). Red represents pressure, blue is air motion amplitude. (From http://www.phys.unsw.edu.au/jw/brassacoustics.html)

Thus a conical pipe stopped at one end and open at the other produces a full set of harmonics. Note that an instrument based on a cone cannot be completely closed as it would be impossible to blow into! It can, however, be shown that the addition of a mouthpiece ‘completes’ the truncated cone by the additional volume of air in the mouthpiece [Campbell/Greated, 1987, p. 310].

3.4 Input impedance

For the simple pipes shown the resonances can be predicted relatively easily. For a brass instrument, where the pipe consists of a combination of conical, straight and flared sections of pipe, it can be difficult to predict the resonances mathematically and measurements may need to be taken. A convenient objective measure to describe the response of an instrument is the acoustic input impedance. This is the relationship between the volume flow velocity of the air blown into the mouthpiece and the resulting sound pressure, and is defined as:

$$Z_{in}(\omega) = \frac{P(\omega)}{U(\omega)}$$

where:

$Z_{in}(\omega)$ is the acoustic input impedance (Ohms)

$P(\omega)$ is the sound pressure (Pa)

$U(\omega)$ is the volume velocity (m^3/s)

Brass instruments have a natural tendency to produce notes when the mouthpiece pressure is high (see Section 3.5), and therefore the acoustic impedance is high at frequencies where notes are produced. The graph below shows the input impedance of a bass trombone and several notes corresponding to the impedance peaks.

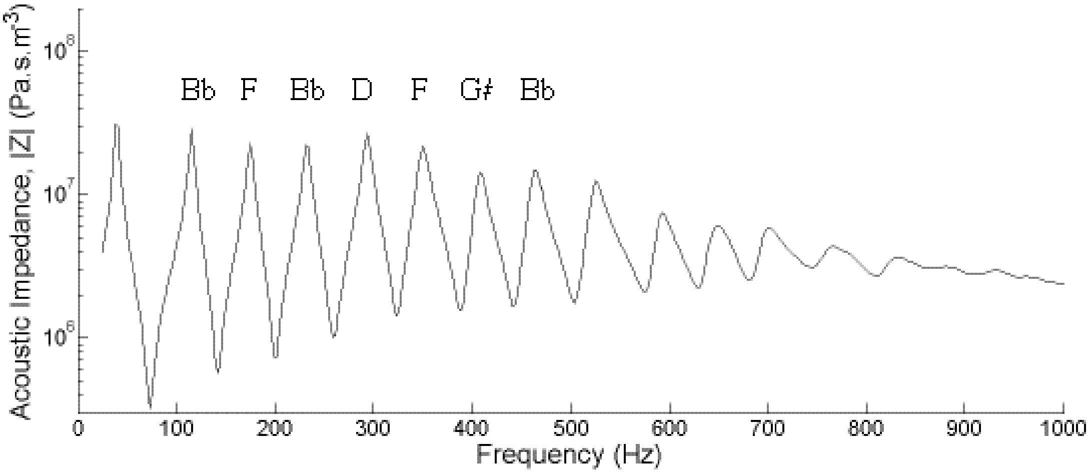


Figure 12 - Bass trombone impedance spectrum (from http://www.phys.unsw.edu.au/jw/brassacoustics.html)

Benade describes a experimental mechanical method of measuring the input impedance of a brass instrument using a pump, capillary tube and microphone:

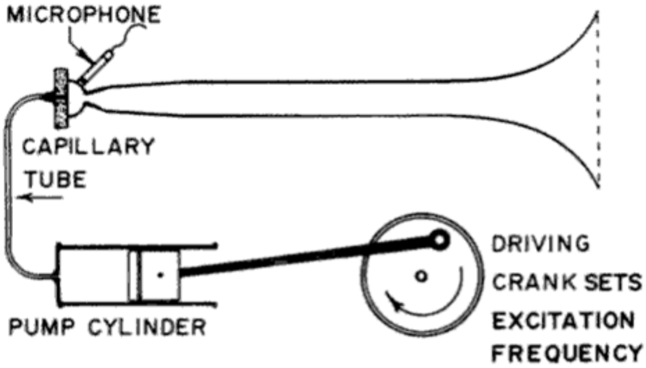


Figure 13 - Measuring input impedance (from Benade, 1990)

The acoustic impedance may therefore be derived mathematically using the pressure measured by the microphone and the volume velocity produced by the pump.

3.5 Producing the vibrations - the function of the lips

Simply blowing down the instrument would produce no sound at all – there would be no vibrating source to start and sustain an oscillation. A wind instrument therefore requires a continuous vibrating source of air to set up and sustain standing waves. On woodwind instruments this is accomplished by a vibrating cane reed, and on brass instruments this is accomplished by the vibrating lips – or in other words, a 'lip reed'.

The lips have mass and are springy so, as when blowing a raspberry, they will naturally open and close, primarily due to the Bernoulli effect [Howard/Angus, 2004, 189]. When required to play a brass instrument this crude raspberry becomes a finely tuned vibration, admitting puffs of air at carefully timed intervals to sustain the oscillation of the air column. The lip is working as a pressure-controlled valve – when the pressure at the mouthpiece is high, the lips naturally open and admit another puff of air timed to reinforce the oscillation. The effect is analogous to pushing a child on a swing, but another more elegant analogy is outlined by Benade by the 'water trumpet', shown in Figure 14.

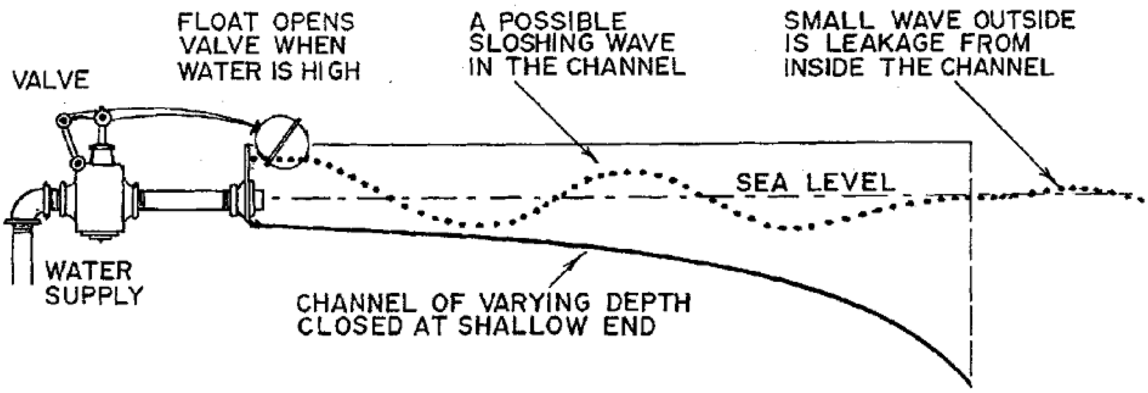


Figure 14 - Benade's 'Water trumpet' (from Benade, 1990)

It can be shown that the movement of the water sloshing longitudinally backwards and forwards in a channel of varying depth obeys the same equations as the movement of the air column in a brass instrument [Benade, 1973]. The height of the water is analogous to the air pressure in the instrument. When the water level (sound pressure) is high, the water valve (lips) is opened and produces a squirt of water (puff of air), sustaining the oscillation.

The behaviour of the lip is therefore more complicated than the crude raspberry. In effect, the lip locks onto the pressure variations at the mouthpiece, synchronising their rate of opening with the resonant frequencies of the instrument to produce a note. This is a positive feedback system at work, which is a fundamental feature of many oscillating systems.

3.6 Non-linearity of the lip vibration

It can be shown by experimentation that the opening and closing of the lips is not sinusoidal, but produces a complex waveform enriched with several harmonics, or *partials*, which are harmonically related to the fundamental. The relative balance of partials determines the *timbre* or tone quality of an instrument. As the notes get higher in pitch, the oscillation becomes increasingly sinusoidal.

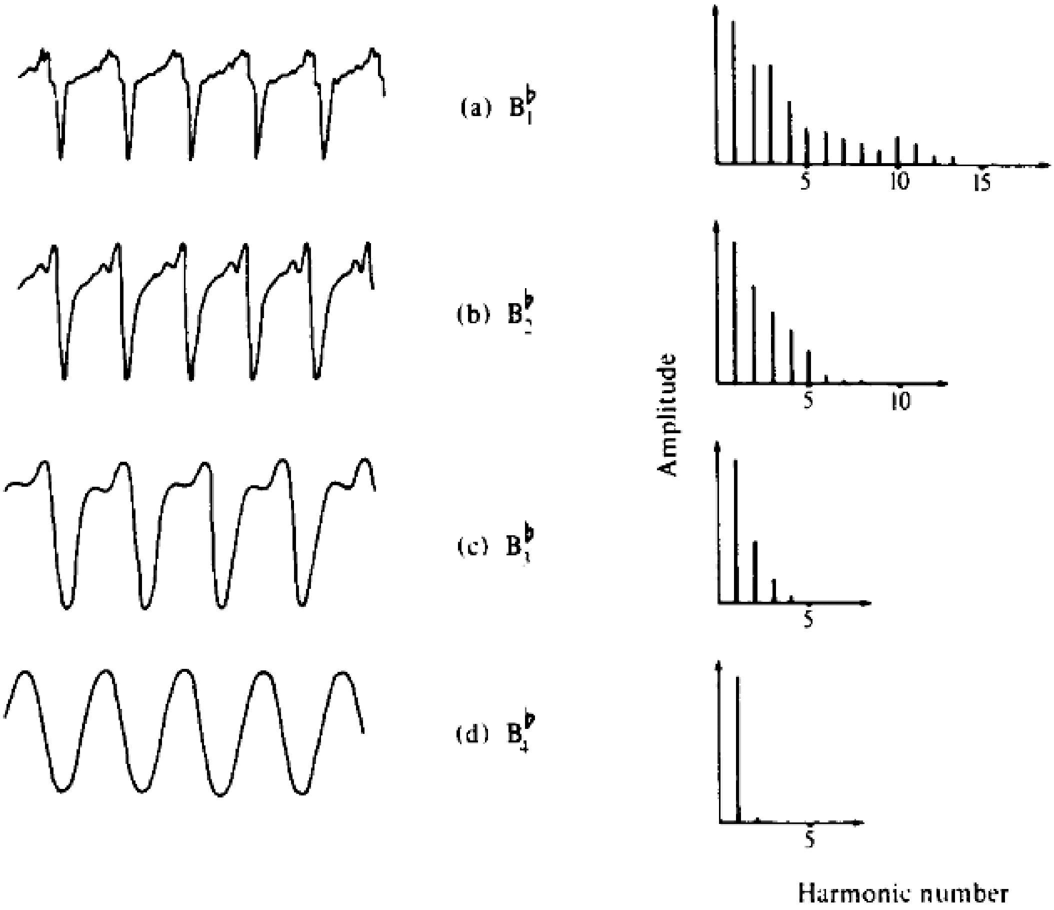


Figure 15 - Waveforms (left) and harmonic spectra (right) of the pressure variations in a trombone mouthpiece during the playing of four notes (from Campbell/Greated, 1987)

3.7 ‘Regimes’ of oscillation

For a single pure tone the pressure-controlled valve described would work satisfactorily, but the note would be reliant on a single resonance to sustain itself and would be thin in sound and difficult to pitch and maintain by the player. In real instruments each partial harmonically related to the fundamental also stimulates the pressure-controlled valve operation, and hence helps to sustain the oscillation. This is referred to by Benade as a *regime of oscillation*. The result is that notes rich in partials are easier to start and maintain than notes with fewer partials. It is interesting to note that higher note on brass instruments have fewer partials than lower notes, and this is one of the reasons that they are more difficult to pitch.

In order to provide a strong regime of oscillation, which in turn produces a strong and easily-controlled note, it is therefore necessary for the instrument to provide a convenient set of resonances which are harmonically related to the fundamental. This essentially means that an effective instrument should ideally produce a full set of harmonically-related resonances. The input impedance spectrum for a strong regime of oscillation from an alphorn, where the harmonics are multiples of 52 Hz, is shown in Figure 16.

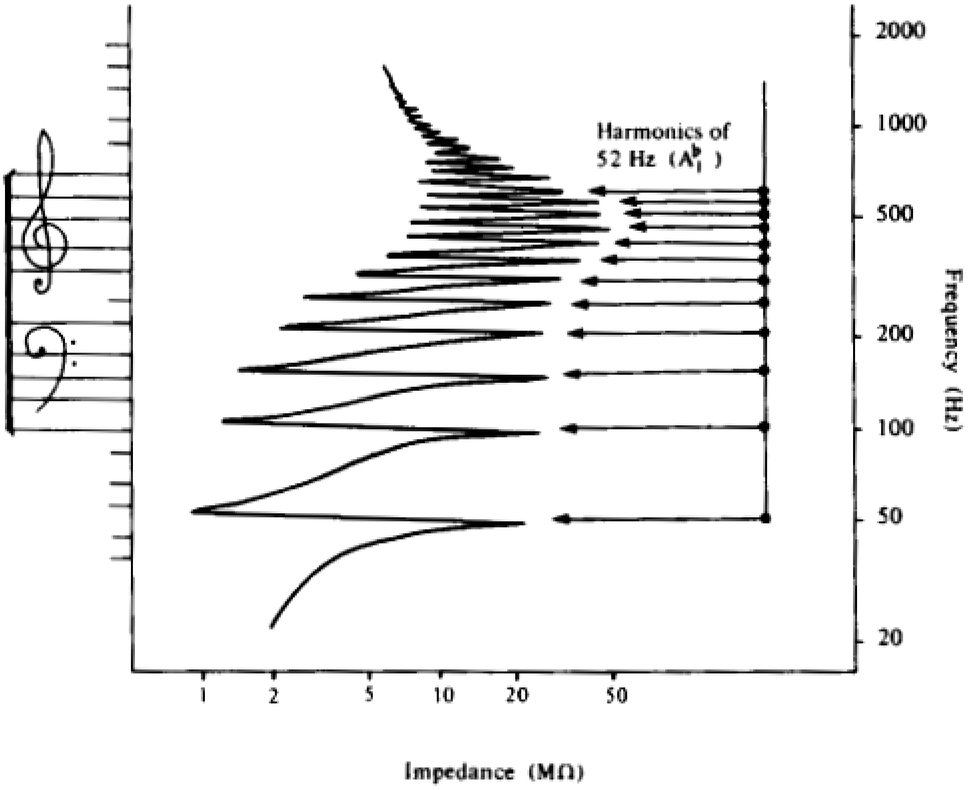


Figure 16 - 'Regime of oscillation' (from Campbell/Greated, 1987)

3.8 Pedal notes

Interestingly, the lowest note playable on many brass instruments is not actually the fundamental but a ‘false’ resonance known as the *pedal note*. This note is not actually a natural harmonic, but is produced by the higher harmonics ($2f$, $3f$, $4f$) combining to help the lips establish a nonlinear vibration at the pedal note frequency. This is a heterodyne tone formed by the difference frequency between adjacent resonances. Heterodyning occurs when two frequencies f_1 and f_2 are input to a nonlinear system, and causes vibration components to occur at (f_1 - f_2) and (f_1 + f_2). In the case of the pedal note, there are many adjacent resonances where the frequency difference between them equals the pedal note frequency.

Figure 17 shows the input impedance graph of a B♭ tenor trombone, where the impedance peaks (to the right of the diagram) correspond to the natural resonances of the instrument. The graph shows how a pedal note with frequency of 58 Hz can be played on a trombone even though there is no resonance at that frequency. The higher harmonics set up a co-operative ‘regime’ which enables the lower note to sound.

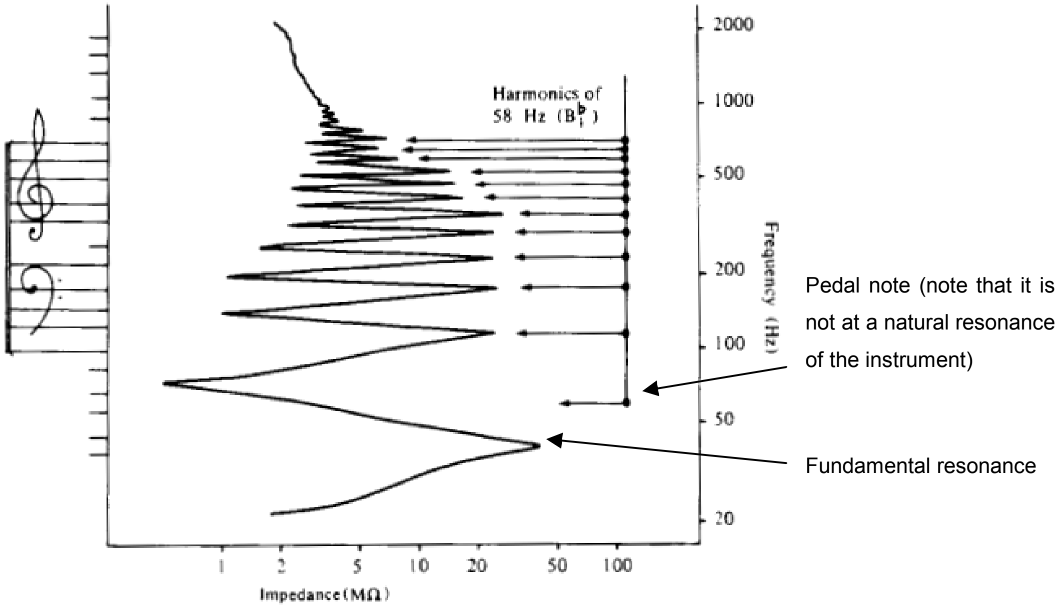


Figure 17 - Impedance curve for B♭ tenor trombone in first position playing the pedal note at 58 Hz (from Campbell/Greated, 1987)

The fundamental of many brass instruments is too low and out of tune to be of musical use, and is generally unplayable by the lip. The pedal note is therefore effectively the ‘first harmonic’ of these instruments.

3.9 Equivalent cone length

It has been established that a usable brass instrument needs to have an air column whose harmonic resonances approximate a complete harmonic series in order to produce a strong ‘regime of oscillation’ (see Section 3.7). A conical instrument naturally meets this requirement, and for this reason it is therefore convenient to refer to the *equivalent cone length* of an instrument as a useful reference for tubes of more complicated profiles. In general, if the *nth* resonance of an instrument occurs at frequency f_n, the equivalent cone length at that frequency is given as:

$$L_e = \frac{nc}{2f_n} \quad \text{[Campbell/Greated, 1987, p. 337].}$$

where:

L_e is the equivalent cone length (m)

n is the harmonic number

c is the speed of sound (m/s)

f_n is the frequency of the resonance (Hz)

Figure 18 shows the equivalent cone length of the harmonics of a B♭ tenor trombone. The diagram shows that, with the exception of the lowest resonance, the instrument has an equivalent cone length of approximately 2.9 metres.

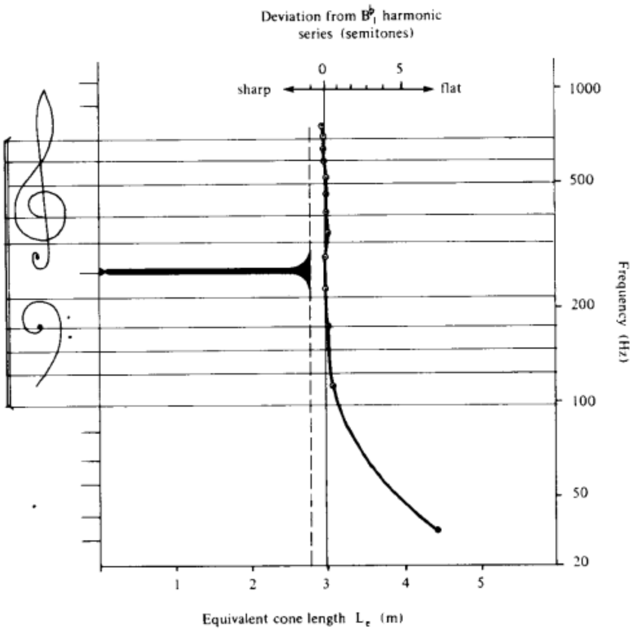


Figure 18 - Equivalent cone length of B♭ tenor trombone (from Campbell/Greated, 1987)

The actual length of the trombone in this case is 2.77 m from end to end, and is indicated by the dashed line on the graph. For frequencies above the first resonance, the trombone behaves like a cone of length 2.9 m.

The equivalent cone length diagram is useful in showing the pitch of a given harmonic against the corresponding harmonic for an ideal conical instrument. Harmonics to the left of the line are sharp (higher in pitch) relative to the cone, to the right of the line are flat (lower in pitch).

Some brass instruments have a predominantly conical bore (for example, the tuba and flugel horn), but most (including the horn) have a conical mouthpipe, significant cylindrical section and a flare and bell. It is interesting to note that brass instruments with large sections of cylindrical pipe also produce a harmonic series that approximates that of a cone, the reasons for which will be discussed later in this project.

The equivalent cone length of a cylindrical tube is shown in Figure 19, and it can be seen that the lower resonances are much flatter than the required full odd and even harmonic series.

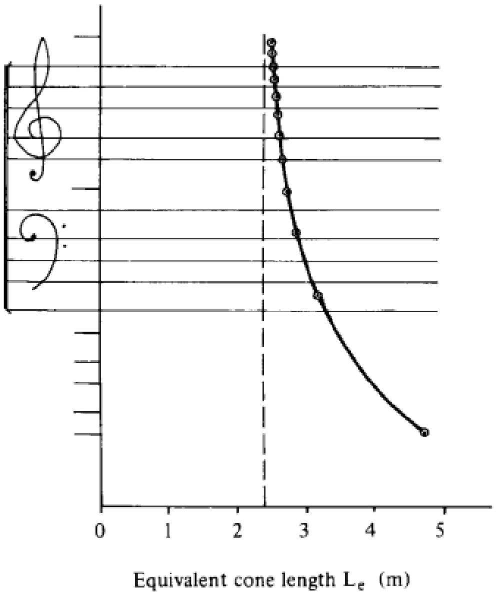


Figure 19 - Equivalent cone length of hosepipe 2.35 metres in length (from Campbell/Greated, 1987)

The instrument maker has therefore over the years adapted the various shapes and proportions of the brass instrument empirically to adjust the harmonic series to that of a cone.

3.10 Sections of the instrument

The horn can be broken down into four main sections – the mouthpiece, mouthpipe, main bore and bell. This is illustrated in Figure 20:

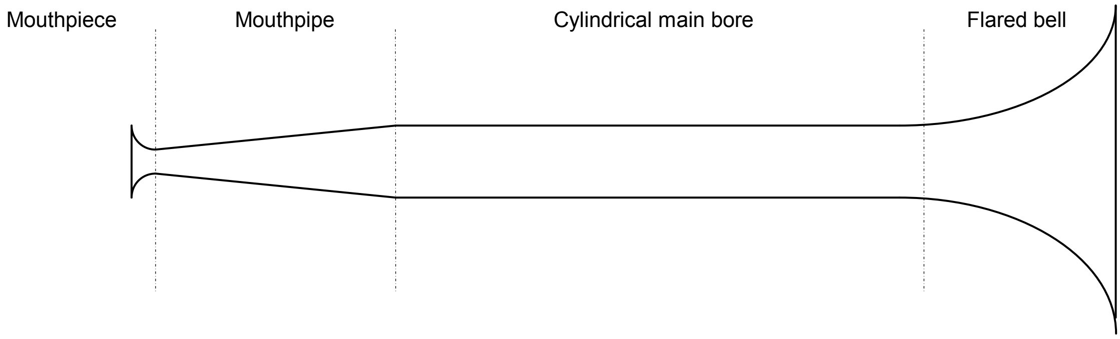


Figure 20 - Sections of the instrument

These sections, and their effect on the instrument, are described individually below.

3.11 Mouthpiece and mouthpipe

The most obvious function for the mouthpiece is to provide a supporting framework for the lips against the end of the instrument. All mouthpieces share common features, but the horn mouthpiece is quite different to that of other brass instruments in that the cup is deeper and more conical. The main features of the mouthpiece are illustrated in Figure 21:

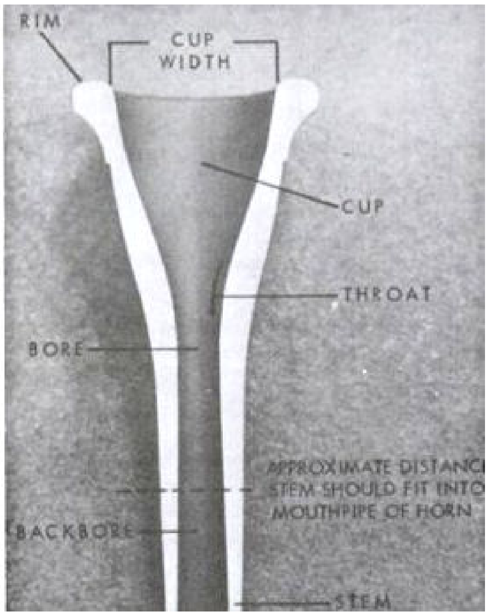


Figure 21 - Diagram of horn mouthpiece (from Farkas, 1956)

The cup of the mouthpiece leads to a narrow constricted section called the throat, followed by a widening backbore. The mouthpiece is inserted into the tapered mouthpipe, which continues this gradual widening into the main cylindrical section of the instrument. The mouthpiece / mouthpipe combination can effectively be considered as a single conical section commencing from the mouthpiece throat.

The cup and backbore form a Helmholtz resonator, and its natural frequency (the so-called 'pop' tone) can be observed by slapping the mouthpiece cup against the hand. The volume of air in the mouthpiece cup has the effect of increasing the effective acoustic length of the instrument in the higher resonances – generally those above the pop tone. The addition of the conical mouthpipe increases this variation in length, and can spread the effect below and above the pop frequency [Campbell/Greated, 1987, p. 343]. Figure 22 shows this effect for a (a) trumpet mouthpiece without backbore, and (b) the same mouthpiece with a tapering backbore and mouthpipe 300 mm long. f_R is the resonant frequency, or 'pop tone' of the mouthpiece.

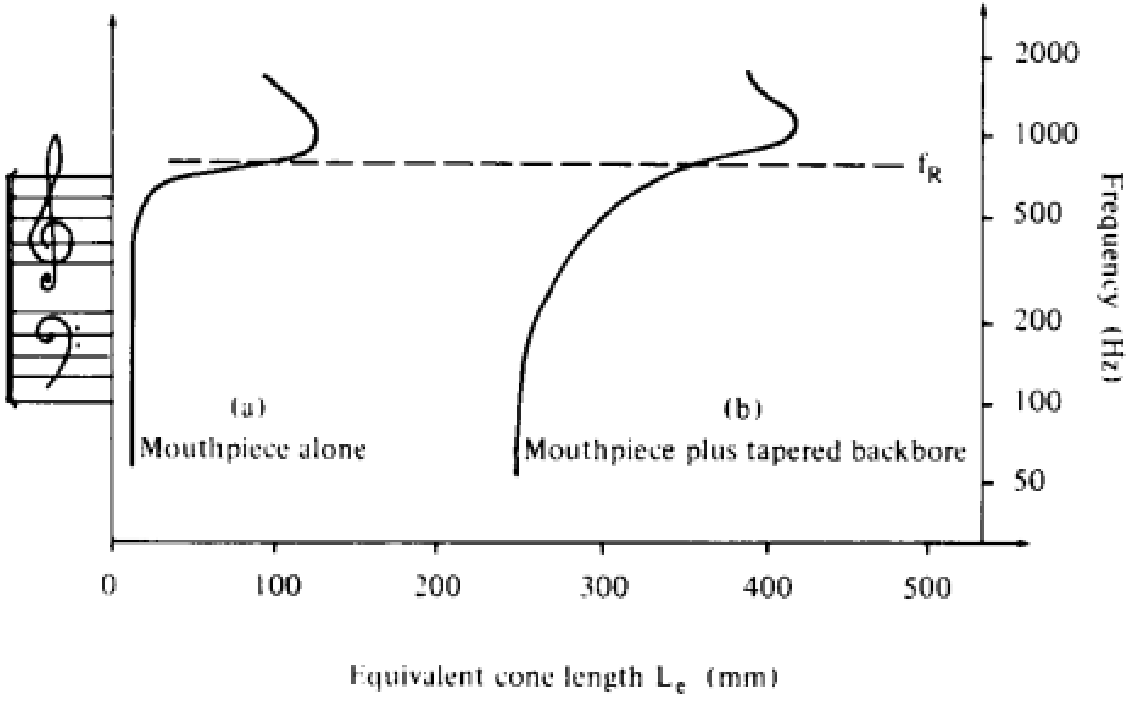


Figure 22 - Effect of trumpet mouthpiece and mouthpipe (from Campbell/Greated, 1987)

What this means is that the pitch of the higher harmonics, which would be rather sharp for a purely cylindrical tube, are flattened to approximate a constant effective length in the usable upper register [Campbell/Greated, 1987, p. 343]. Figure 23 shows the effective cone length for (a) a cylindrical tube closed at one end, (b) the same tube terminated by a mouthpiece cup, and (c) the same tube with a mouthpiece cup, backbore and mouthpipe:

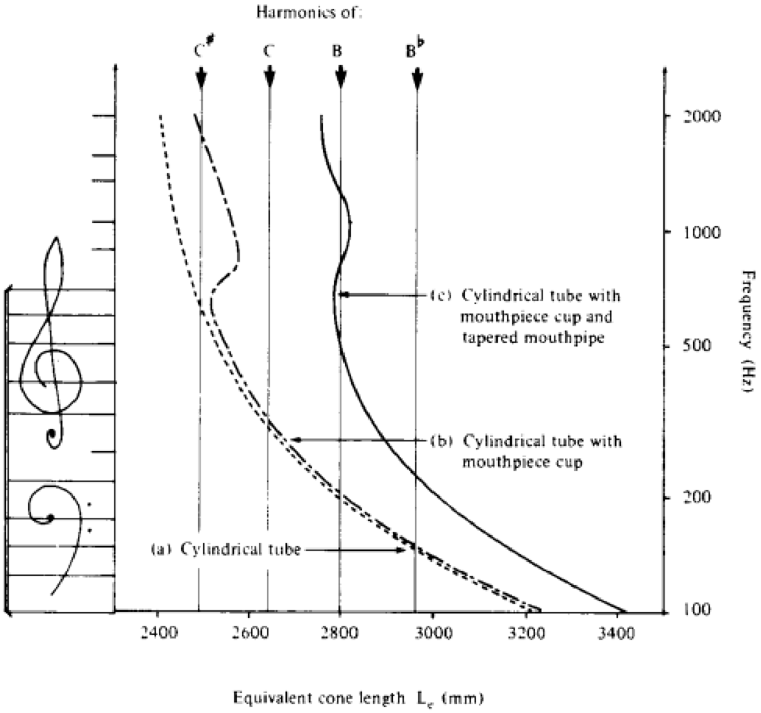


Figure 23 - Mouthpiece / mouthpipe effect on a cylindrical pipe (from Campbell/Greated, 1987)

For the lower resonances, the mouthpiece / mouthpipe has less of a flattening effect than at higher frequencies. The mouthpiece and mouthpipe therefore provide a steady increase in effective length of the instrument from approximately the pop tone to nearly the top of the instrument's playing range [Benade, 1990, p. 416].

3.12 Cylindrical tubing

The cylindrical section of tubing forms the central section of the horn and the principal purpose is to set the lowest pitch of the instrument, as a longer pipe will have a lower fundamental harmonic. It has little effect on the intonation or tone of the instrument. The valves and tuning slides are contained within the cylindrical section.

3.13 Valves / tuning slides

The valves enable short lengths of extra tubing, the *tuning slides,* to be inserted into the main cylindrical section of pipe (see Section 2.3).

The tuning slides will not achieve the exact lowering of pitch of the entire range of the instrument. This is because the relative nature of pitch is logarithmic, so that a percentage change in length is required for a given interval. Therefore, at low frequencies a longer length is required than at higher frequencies to achieve the required lowering interval, so a longer tube would be required for lower resonances than higher.

In practice, for a given instrument each harmonic is not perfectly in tune anyway. The player will use the appropriate fingering to obtain the best intonation for a given note, and there is also some flexibility in ‘lipping’ notes into tune by varying the lip tension.

3.14 Flare / bell

The diameter of brass instruments bells increases at an exponential rate, terminating in a very rapid flare as shown in Figure 24:

Figure 24 - Bell flare

There are two principal effects of the flaring bell:

1. It acts as a gradual impedance transition between the pipe and the atmosphere, rather like an acoustic transformer, and;
2. It affects the tuning of some harmonics in a similar manner to the mouthpipe.

3.14.1 Acoustic transformer

The primary purpose of the bell would appear to be obvious – common sense would dictate that it somehow amplifies the sound from the instrument in the same manner as a gramophone horn or a horn loudspeaker. This is true, but the instrument horn is not designed in quite the same way as a gramophone horn, which is designed to project as much sound as possible. The instrument horn must be designed to reflect most of the wave back into the instrument to produce the standing waves required to produce notes at the harmonic resonances.

Any exponential horn will naturally have a *cut-off frequency* below which it will be a very inefficient radiator of sound. Below the cut-off frequency most of the sound is reflected back into the instrument. This is of critical importance in horn loudspeaker design, where maximum radiation efficiency is required over a given bandwidth, and hence the cut-off frequency is designed to be lower than the minimum reproduced frequency. A brass instrument therefore produces most notes at frequencies below the cut-off frequency, as the reflections are required to produce strong standing waves. A balance must be struck by the instrument designer to produce a bell that allows enough sound to be reflected back into the instrument to produce strong resonances at the required frequencies, but enough sound out of the instrument to make a sufficiently loud noise. In fact, only a very small proportion of the sound ‘leaks’ out of the bell. Sound pressure levels up to 175 dB re 20μPa have been measured inside an instrument [Noreland, 2003, p. 8], but a horn is clearly not capable of producing audible sound at that level.

Waves with a wavelength comparable with the curvature of the bell are much more readily radiated then those with a long wavelength [Wolfe]. This means that the horn is a much more efficient radiator of sound at these high frequencies, but this has the downside that the standing waves are reduced in amplitude and the higher resonances are therefore more difficult to produce than the low. This is another reason why high notes are relatively difficult to play (see Section 3.6 for another related reason).

3.14.2 Change in effective length of the instrument

The second effect of the bell is less obvious. As discussed in Section 3.9 the mouthpipe changes the pitch of the upper harmonics, and the bell has a similar effect on the lower harmonics. This is related to the effective ‘acoustic length’ of the instrument, which varies with frequency. In simple terms, low frequencies are reflected at the narrower part of the bell, and high frequencies are reflected at the wider flare. It is possible to prove this mathematically with various solutions of the wave equation, but to do so is complicated and beyond the scope of this project. In summary, it can be shown that the air in a flared pipe behaves as a dispersive medium [Kinsler/Frey, 2000, p. 415] and that the speed of sound increases with distance into the flare in a frequency-dependent manner. Conversely, in a cylindrical pipe with straight sides the wave travels at the same speed as in the open air.

For waves above the cut-off frequency of the horn most of the sound passes into the open air, and it effectively acts as a simple megaphone. Below the cut-off frequency, depending on the frequency of the wave, a point is reached in the bell flare where the speed of sound becomes infinitely large [Campbell/Greated, 1987, p. 345]. At this ‘turning point’ the wave is reflected back into the instrument. The range in which this occurs is sometimes referred to as a ‘forbidden zone’, as the sound wave is unable to ‘tunnel’ through it. This is illustrated in Figure 25 for a B♭ tenor trombone. It can be seen that the lower modes are reflected earlier in the bell flare than the higher, and the height of the forbidden zone represents the cut-off frequency.

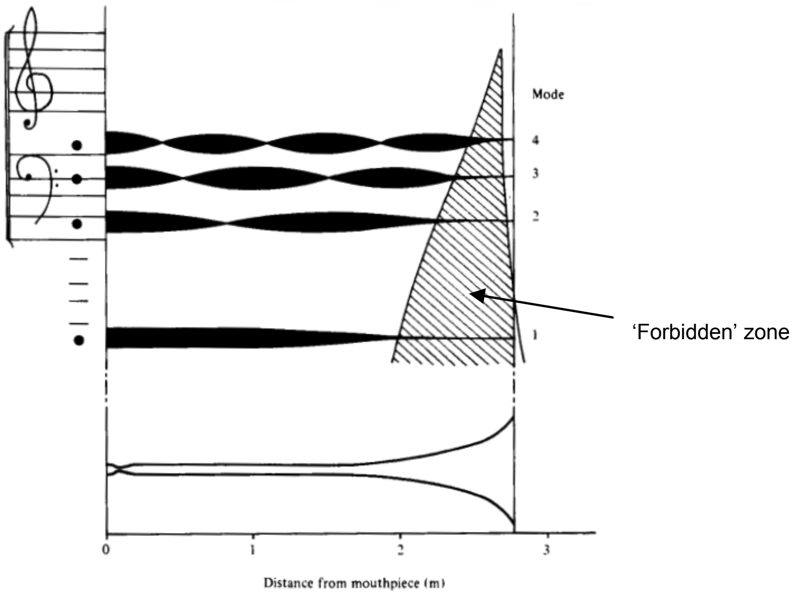


Figure 25 - 'Forbidden zone' for a B♭ tenor trombone (from Campbell/Greated, 1987)

In effect, the instrument behaves like a short pipe at low frequencies and a long pipe at high frequencies. Figure 26 shows the effect on the pressure distribution of the lowest three modes as modified by the flaring bell. As the wave velocity increases, the shape of the wave loses the sinusoidal shape and becomes more stretched out towards the end of the bell. The diagram also shows the corresponding cylinder with a length which gives an equal natural frequency for each mode, and the dotted lines indicate the nature of the standing wave patterns in that cylinder.

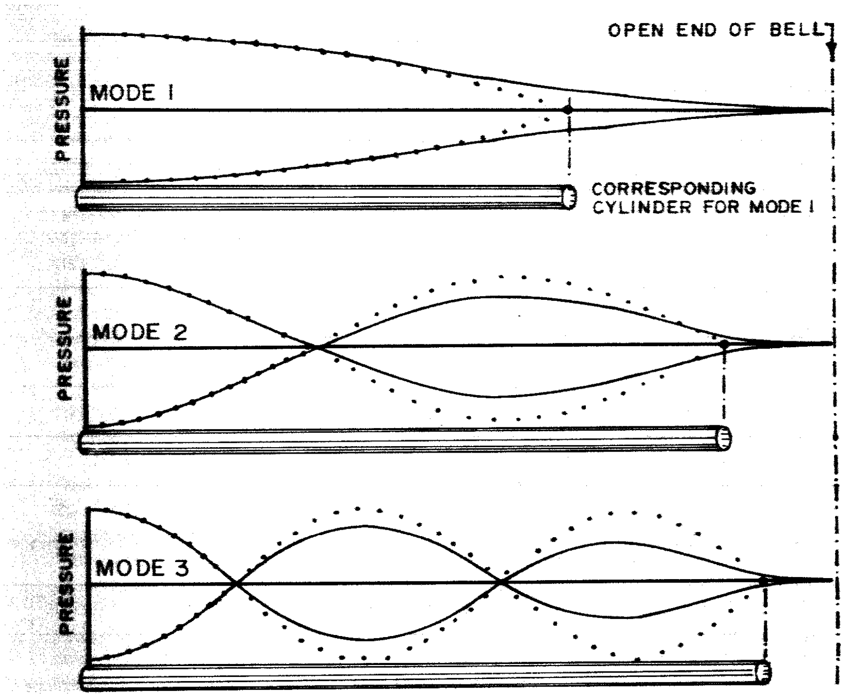


Figure 26 - 'Turning point' of wave at bell end (From Benade, 1990)

The effect of the bell on the lowest resonances is therefore to raise their pitch. In practice, the fundamental cannot be raised to a useful frequency, and the flare is chosen to bring the resonances from the 2nd upwards to an approximate harmonic relationship [Campbell/Greated, 1987, p. 345]. This is illustrated in Figure 18 for a trombone. Note that the diagram shows that the fundamental does not fall into line with the rest of the harmonics on the constant equivalent cone length line.

3.14.3 *Directivity and radiated spectrum*

The bell also dictates the directivity and radiated frequency response of the instrument. At low frequencies the instrument will tend to be less directional due to diffraction effects, but will become increasingly more directional at higher frequencies.

3.14.4 *Purpose of the hand*

The horn is unique in having the player's hand inserted into the bell. This peculiarity originated from manipulating the pitch of the open notes (see Section 2.1.2), but is still used today as it influences the tone of the instrument and the player can correct the intonation of notes by moving the hand. Placing the hand in the bell will make it easier to play higher notes due to the additional reflection of the standing wave, but it will also affect the timbre by reducing the amplitude of higher frequency notes. Figure 27 shows the effect of inserting the hand into the bell on the acoustic impedance curve of the horn:

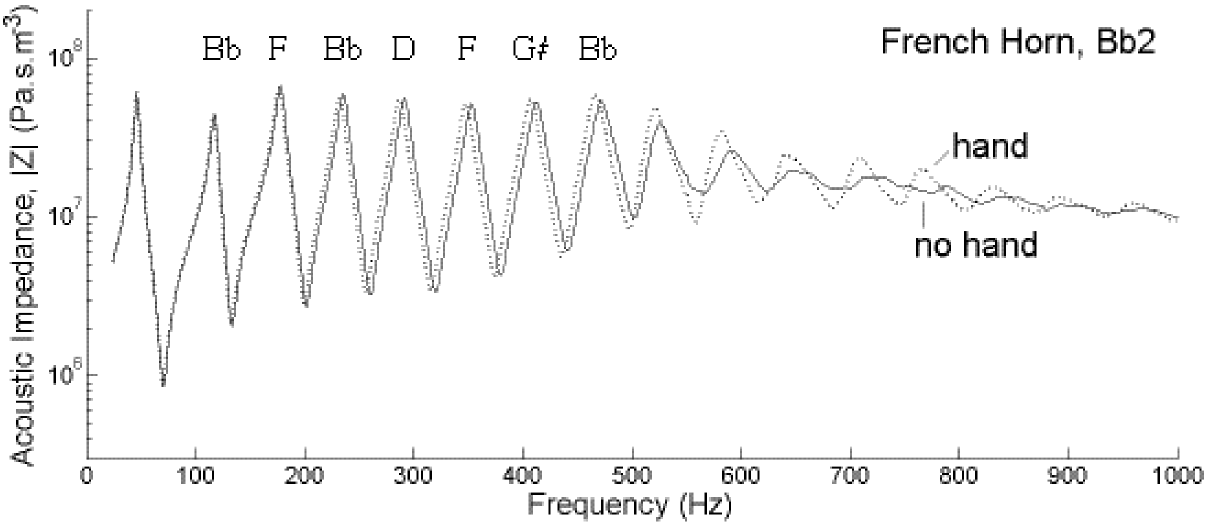


Figure 27 - Impedance curve of horn with and without hand in bell (from http://www.phys.unsw.edu.au/jw/brassacoustics.html)

The diagram shows that introducing the hand into the bell increases the impedance peaks of the higher resonances, which makes the higher notes easier to play.

4 EXPERIMENTS

4.1 Overview

The aim of the experiments is to compare the harmonic series produced by the horn with that of a length of cylindrical open-closed pipe.

As discussed in Section 3.10, the horn bore consists of four distinct sections:

1. A mouthpiece;
2. A conical mouthpipe, approximately 500 mm in length from the end of the mouthpiece to the first tuning slide. For the purposes of this experiment, the mouthpiece and mouthpipe will be considered as a single section;
3. A cylindrical section which varies in length depending on whether the B♭ or the F side of the instrument is being used. For this experiment the B♭ side was used;
4. A flaring horn terminating in a bell, approximately 1500 mm in length from the end of the cylindrical section to the end of the bell.

4.2 Methodology

A length of hosepipe of ½” diameter was obtained, which is similar in bore size to that of the instrument. A bicycle handlebar grip was placed over the end to allow easier coupling of the pipe to the lips and an airtight seal was made with self-amalgamating tape. In order to study the effect of each section separately, two lengths of hosepipe were cut - one to replace the mouthpiece and mouthpipe, and the other to replace the bell. The hosepipe could be ‘plumbed in’ to the horn by removing tuning slides and connecting it to the horn tubing. The lengths of the pipes were cut to approximately the same length as the sections of horn they replaced.

The modified instrument is shown in Figure 28, showing the sections of horn tubing 'replaced' with hosepipe:

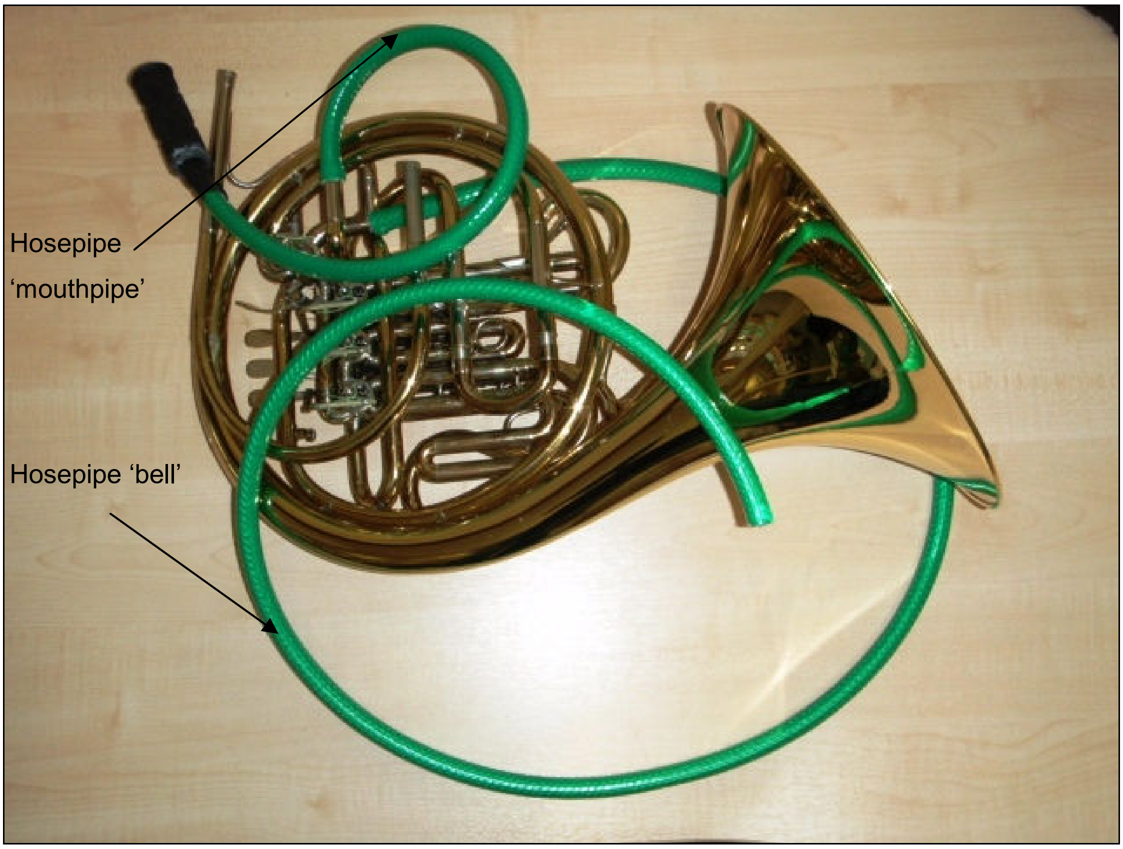


Figure 28 - Modified horn

The following combinations of pipes were used:

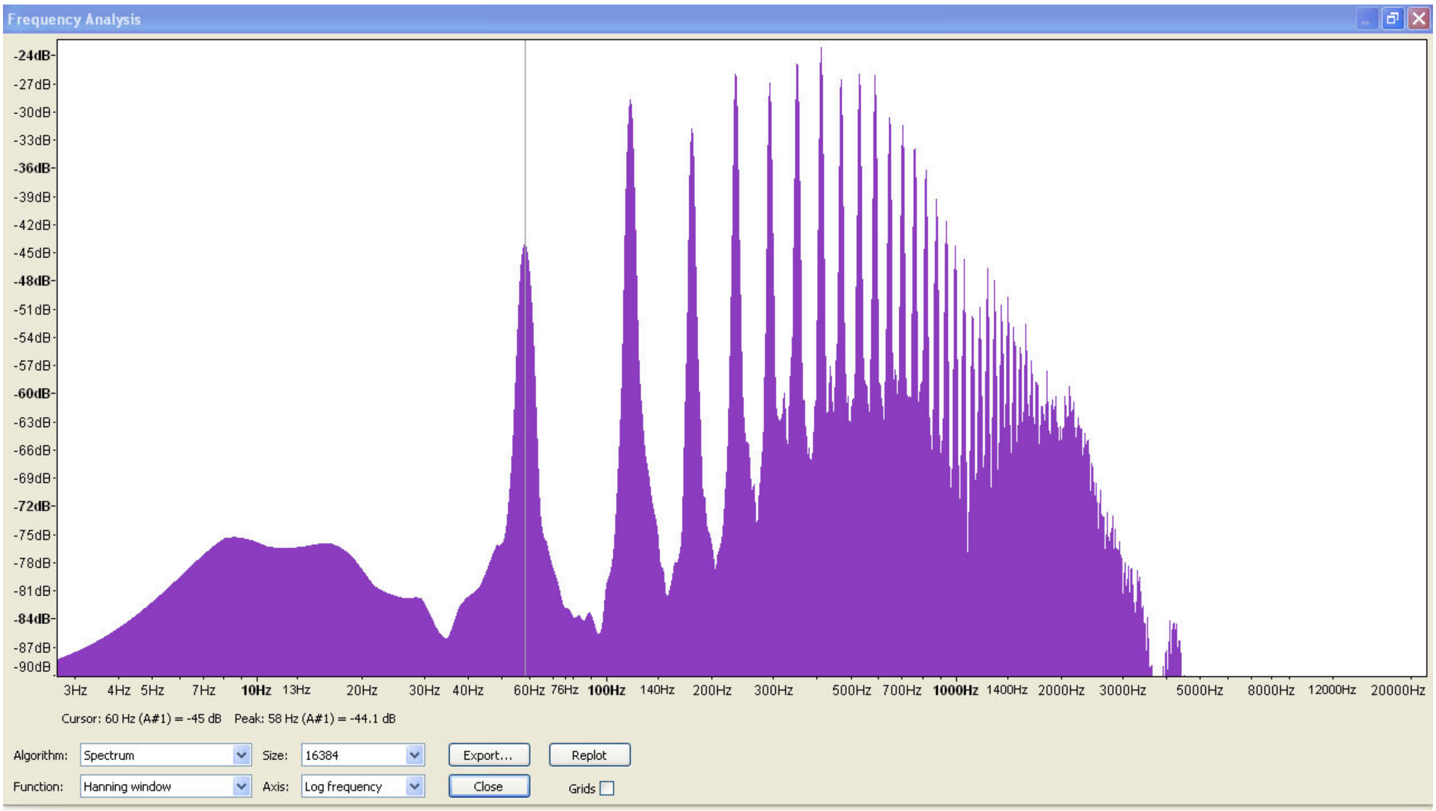
1. The unmodified horn
2. Hosepipe 'mouthpipe' and hosepipe 'bell' (this should approximate a cylindrical pipe, as the central section of horn piping is cylindrical)
3. Hosepipe 'mouthpipe', horn bell (this should study the effect of the horn bell on a cylindrical pipe)
4. Horn mouthpipe, hosepipe 'bell' (this should study the effect of the horn mouthpipe on the cylindrical pipe).

Note that for measurements using the horn bell the hand was placed in the bell as for normal playing. It was attempted, as far as possible, to play each harmonic 'neutrally' – that is, not to adjust the tuning of harmonics with the lip or hand[1].

For each pipe combination the natural harmonics were played from the lowest to the highest possible.

4.3 Measurement equipment and analysis

To analyse the frequencies of the harmonic series, a laptop, good-quality sound card and microphone were used to make recordings with the freeware 'Audacity' digital audio editor. Audacity allows the user to perform FFT analysis on sound samples, and this enabled narrow-band analysis of the spectrum so that individual harmonics could be identified. A screenshot from Audacity showing the lowest playable note on the horn is shown in Figure 29:



Cursor: 60 Hz (A#1) = -45 dB Peak: 58 Hz (A#1) = -44.1 dB

Algorithm: Spectrum
Size: 16384
Function: Hanning window
Axis: Log frequency

Figure 29 - Plot of spectrum for second pipe resonance

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[1] When playing the instrument normally, it is a natural musical process for the player to correct tuning and intonation by 'lipping' the note into tune, or adjusting the hand position. To produce an objective study of the natural resonances it would be important not to do this.

The spectrum shows many peaks which form the fundamental and harmonic resonances above (partials). The frequency of the note played was taken to be the lowest frequency peak.

The following information was gathered for each ‘instrument’:

- The playing characteristics of the instrument were observed;
- The frequencies of each harmonic were noted from the lowest to the tenth resonance. Note that in some cases higher resonances were playable, but their frequencies were found to be somewhat variable, and hence unreliable;
- The equivalent cone length was calculated for the series of resonances, as this allows the deviation from an ‘ideal’ instrument with a full harmonic series to be observed. See Section 3.9 for an explanation of equivalent cone length.

The results are presented in this section and further analysis is undertaken in Section 5.

4.4 Results – horn

The unmodified horn was analysed first. The instrument was played in the conventional manner with the hand in the bell.

4.4.1 Playing characteristics

As would be expected from a good instrument, the horn played well with easily pitched resonances in tune. Notes were playable from low F below the bass clef to E above the treble clef (in the horn pitch of F), but the higher notes were unstable as they are out of the normal range. The harmonic series is reproduced on track 6 of the CD.

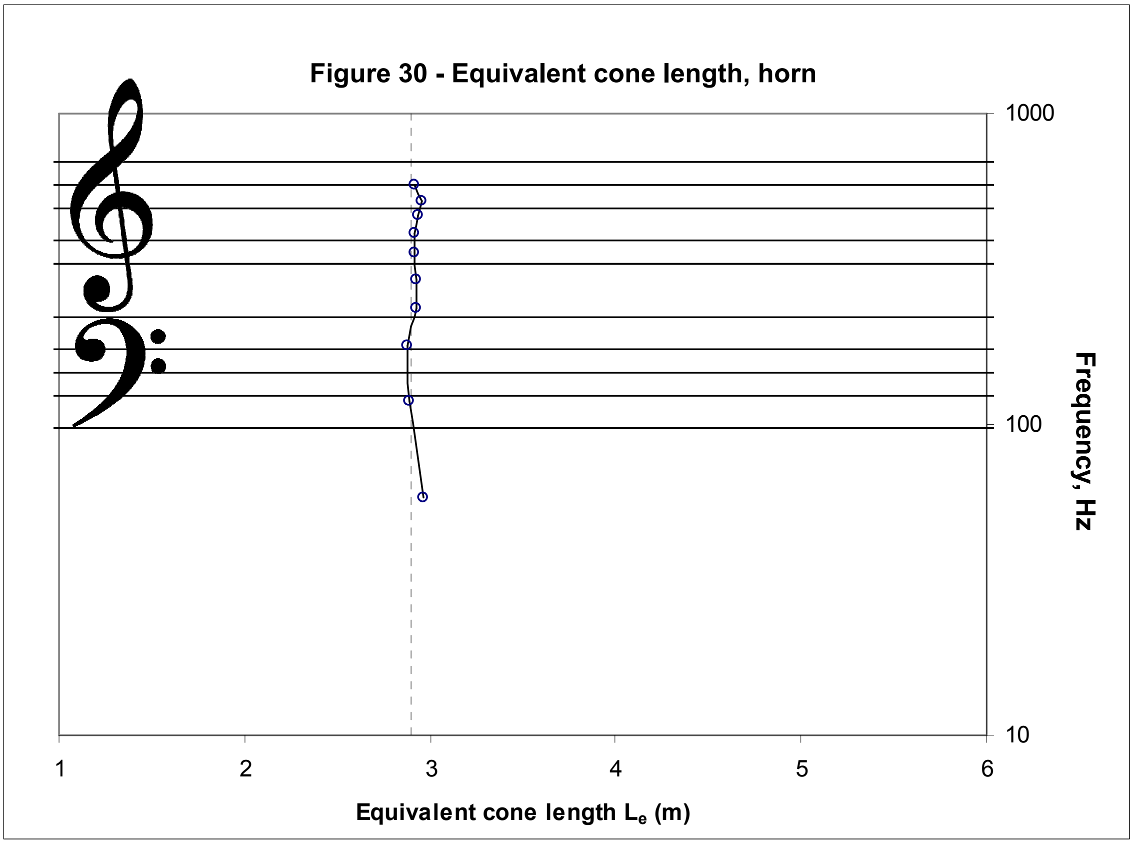
4.4.2 Harmonic series

The frequencies of the harmonic series are shown in Table 1 along with the approximate concert pitch:

Table 1 - Horn harmonic series

Harmonic no.	Frequency, Hz	Note (concert pitch)
1	58	B♭1
2	119	B♭2
3	179	F3
4	235	B♭3
5	294	D4
6	354	F4
7	413	A♭4
8	469	B♭4
9	524	C5
10	590	D5

The equivalent cone length is illustrated in Figure 30:



The graph shows that the horn has a very close relationship to harmonics of a cone of length 2.9 m. As discussed in Section 3.6 of this report, to be musically useful a brass instrument needs to be able to produce a complete harmonic series in order to play strong and stable regimes of oscillation. The measurements indicate that the horn produces this well, as one would expect.

4.5 Results – hose mouthpipe / hose bell

The main and B♭ tuning slides were removed and the hose ‘mouthpipe’ and ‘bell’ were attached. In theory the instrument should now approximate a completely cylindrical tube.

Figure 31 - Hose mouthpipe / hose bell

4.5.1 Playing characteristics

It was noted that the instrument was now considerably more difficult to play. It was possible to produce a set of resonances, but the notes were quieter and generally quite difficult to pitch, particularly the very lowest note. The notes were also quite flexible in pitch and could be ‘bent’ upwards and downwards quite easily. It was also noticed that intervals between harmonics were much wider in pitch than normal in the lower register. Higher up the harmonic scale the notes became closer to the expected pitches from the horn, and it was possible to play recognisable tunes using the harmonic series.

The harmonic series can be heard on track 7 of the CD.

4.5.2 Harmonic series

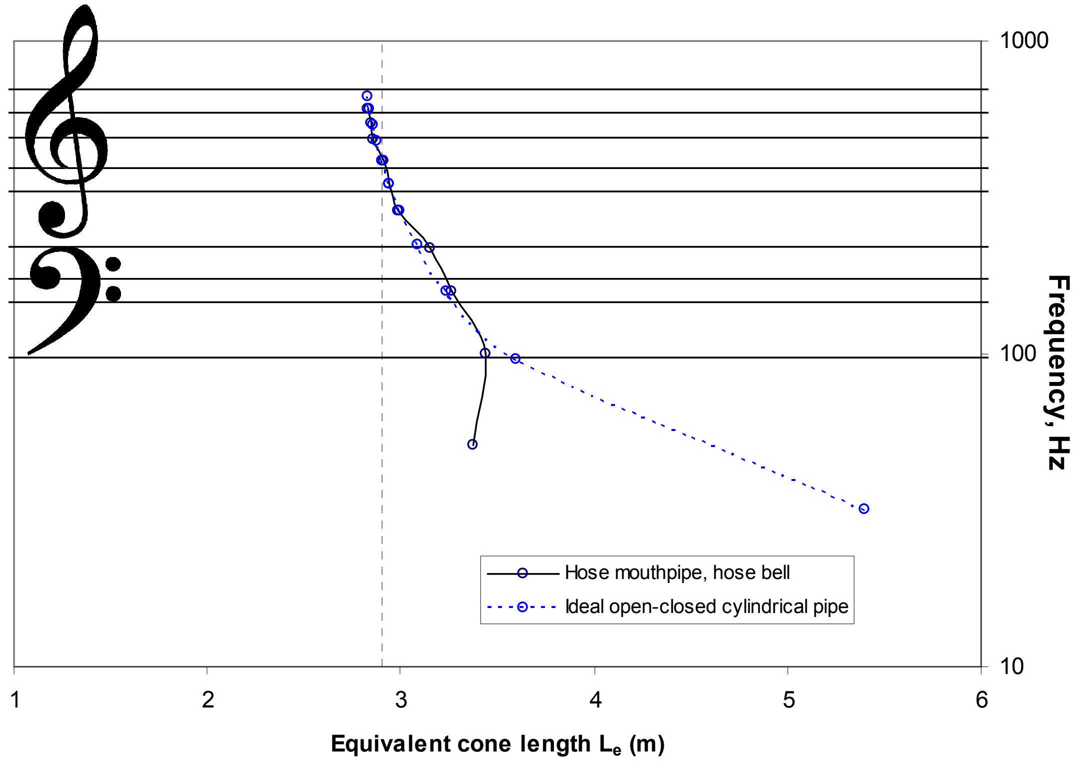
The frequencies of the harmonic series are shown in Table 2, along with the approximate concert pitch. As the instrument should now be a cylindrical pipe, the harmonic series of an open-closed cylindrical pipe 2.7 m long is shown for comparison (2.7 m is chosen as it appears to be the best-fit line to the measured resonances). The resonances are calculated as odd multiples of the ¼ wavelength as described in Section 3.2:

Table 2 - Harmonic series, hosepipe instrument

	Instrument		**Theoretical closed-open cylindrical pipe, 2.7 m long**	
Harmonic no.	**Frequency, Hz**	**Note (concert pitch)**	**Frequency, Hz**	**Note (concert pitch)**
1	51	A♭1	32	C1
2	100	A♭2	96	G2
3	158	E♭3	159	E♭3
4	218	A3	223	A3
5	288	D4	287	D4
6	351	F4	350	F4
7	413	A♭4	414	A♭4
8	482	B4	478	Bb4
9	544	D♭5	541	D♭5
10	608	E♭5	605	E♭5

The equivalent cone length of the instrument and the idealised open-closed cylinder is illustrated in Figure 32:

Figure 32 - Equivalent cone length, hose instrument versus open-closed cylinder



The curves of the open-closed pipe and the instrument follow each other reasonably well from the third resonance upwards, but the second predicated resonance is slightly flat and the first resonance is very flat.

Compared with the equivalent cone length line the instrument is flat at low pitches and sharp above the 8th resonance.

4.6 Results – horn mouthpipe / hosepipe bell

The horn mouthpiece / mouthpipe and hosepipe bell were played. This should effectively evaluate the influence of the horn mouthpipe on the cylindrical pipe.

Figure 33 - Horn mouthpipe / hose bell

4.6.1 *Playing characteristics*

The instrument was much easier to play than the hose mouthpipe / horn bell. The notes were generally easier to ‘centre’ than the hosepipe instrument, but still more difficult to pitch than on the horn. Like the hose mouthpipe / hose bell, the lower harmonics were much flatter than the horn, and the lowest resonance was very difficult to sound and weak in volume. In general the sound was much quieter than a normal instrument.

The harmonic series is reproduced on track 8 of the CD.

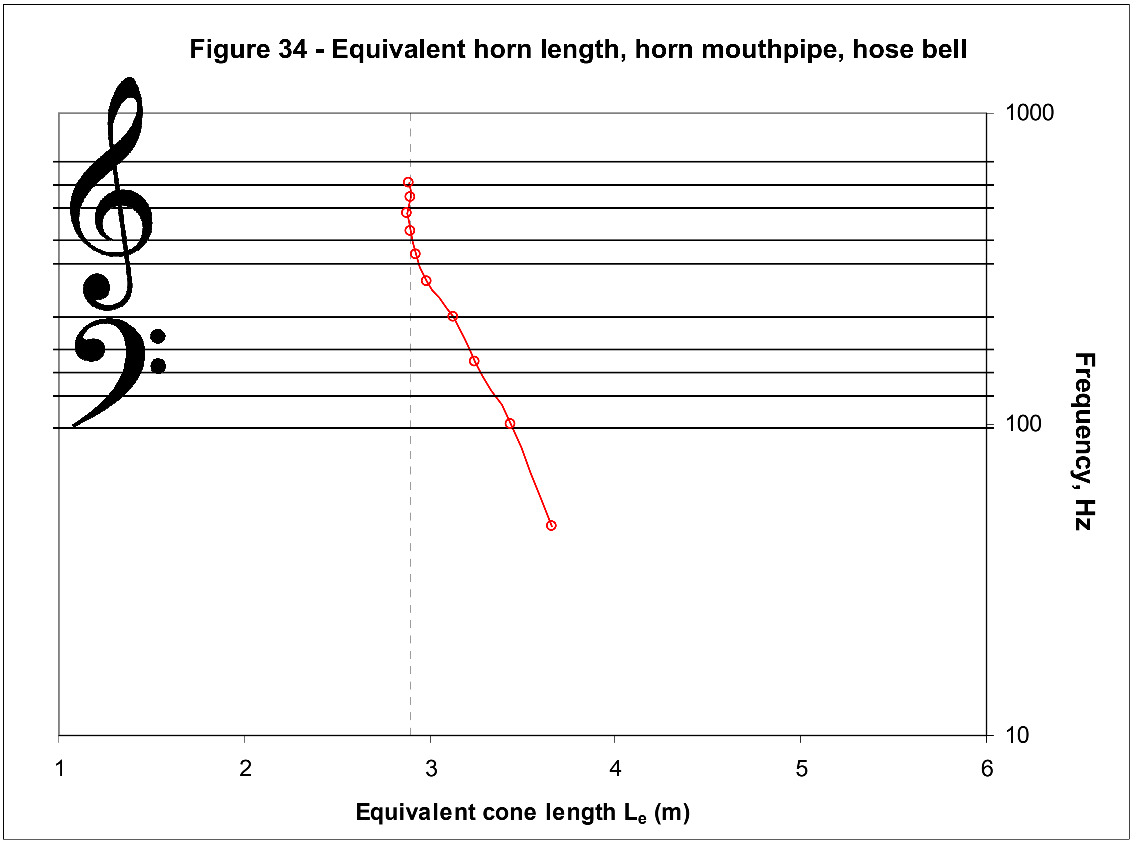
4.6.2 Harmonic series

The frequencies of the harmonic series are shown in Table 3, along with the approximate concert pitch:

Table 3 - Harmonic series, horn mouthpipe / hosepipe bell

Harmonic no.	Frequency, Hz	Note (concert pitch)
1	47	G♭1
2	100	G2
3	159	E♭3
4	220	A3
5	288	D4
6	353	F4
7	416	A♭4
8	478	B♭4
9	535	C5
10	595	D5

The equivalent cone length is illustrated in Figure 34:



The graph shows that the mouthpipe has little effect on the lower resonances, which are still flatter than the constant equivalent cone length. However, the mouthpipe appears to have a flattening effect on the 6th resonance upwards, which become very close to the constant 2.9 m equivalent cone length.

4.7 Results – hose mouthpipe, horn bell

The hosepipe mouthpipe and horn bell were played. This should effectively evaluate the influence of the horn bell on the cylindrical pipe.

Figure 35 - Hose mouthpipe / horn bell

4.7.1 Playing characteristics

The instrument felt much more 'normal' to play than the other hosepipe combinations. The pitches felt much more in tune with the normal instrument, but were more unstable in pitch and difficult to 'centre'.

The harmonic series is reproduced on track 9 of the CD.

4.7.2 Harmonic series

The frequencies of the harmonic series are shown in Table 4, along with the approximate concert pitch:

Table 4 - Harmonic series, hose mouthpipe, horn bell

Harmonic no.	Frequency, Hz	Note (concert pitch)
1	57	A1
2	115	B♭2
3	176	F3
4	236	B♭3
5	296	D4
6	354	F4
7	412	A♭4
8	475	B♭4
9	534	C5
10	621	E♭5

The equivalent cone length is illustrated in Figure 36:

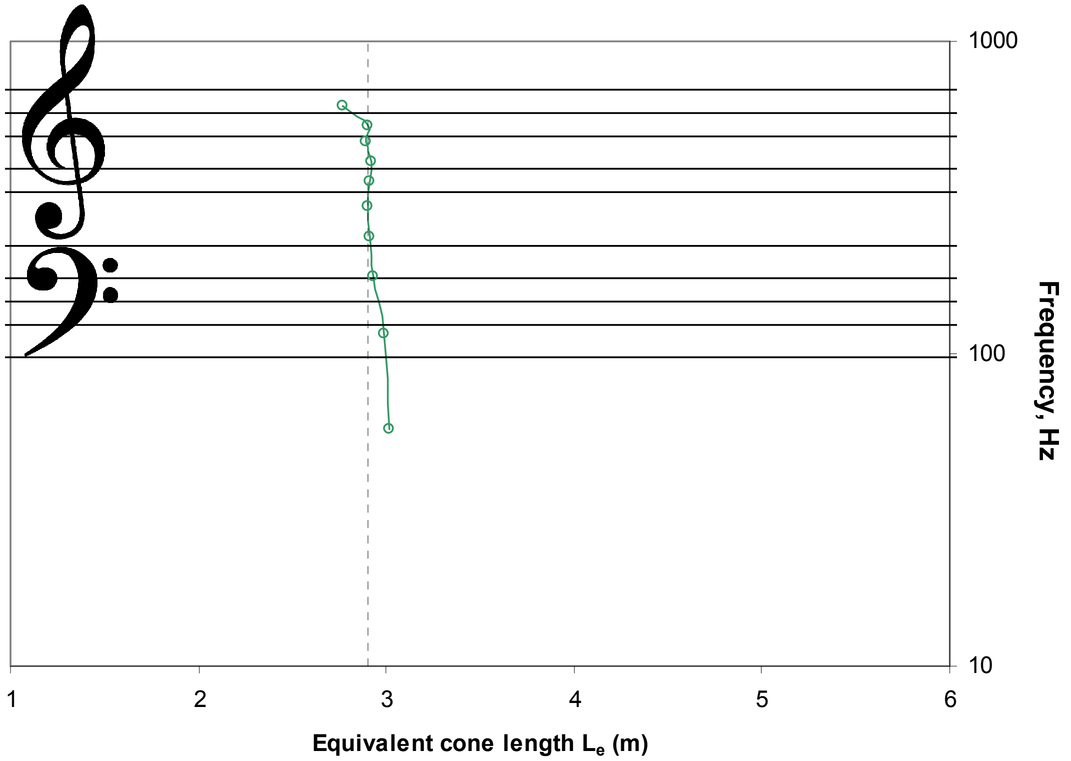


Figure 36 - Equivalent cone length, hose mouthpipe, horn bell

The graph shows that the lower resonances have been sharpened from those of the cylindrical pipe to be quite close to the real instrument. The higher resonances also appear to be flattened to be close to the ideal series, although the 10th resonance is sharp.

5 ANALYSIS

5.1 General overview

The results indicate that the horn mouthpipe and bell appear to affect the tuning of the harmonics in different registers. The graph below compares the equivalent cone length of the three experimental conditions and the theoretical closed-open pipe:

- Hose mouthpipe / hose bell
- Horn mouthpipe / hose bell
- Hose mouthpipe / horn bell
- Ideal closed-open cylindrical pipe 2.7 m long

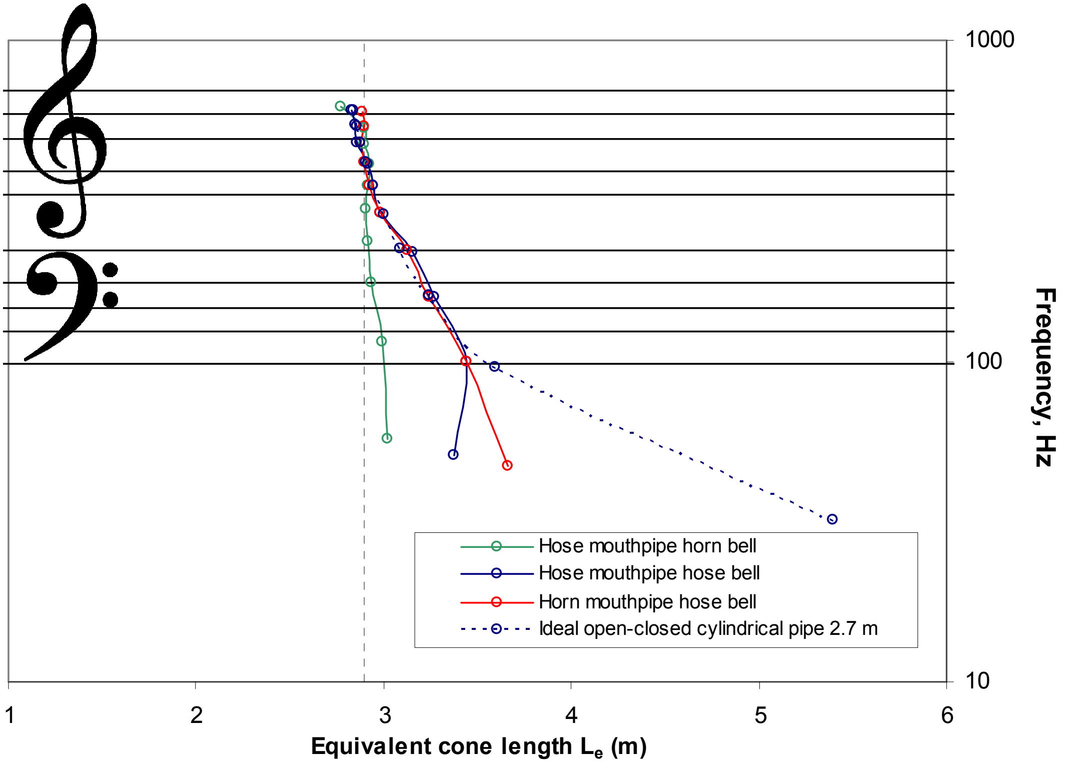


Figure 37 - Comparison of equivalent cone lengths

Some immediate conclusions can be drawn from this graph:

1. The lowest harmonic predicted for an ideal closed-open pipe is very far out from all the measurements.
2. The hose mouthpipe / hose bell and horn mouthpipe / hose bell both follow the ideal closed-open cylindrical pipe from the third resonance upwards reasonably closely.
3. The horn bell sharpens the lower resonances considerably and brings them very close to the ideal constant equivalent cone length.
4. The horn mouthpipe has practically no effect on the lower resonances, but slightly flattens the upper resonances and brings them closer to the ideal cone resonances.

These points will be analysed in more detail below.

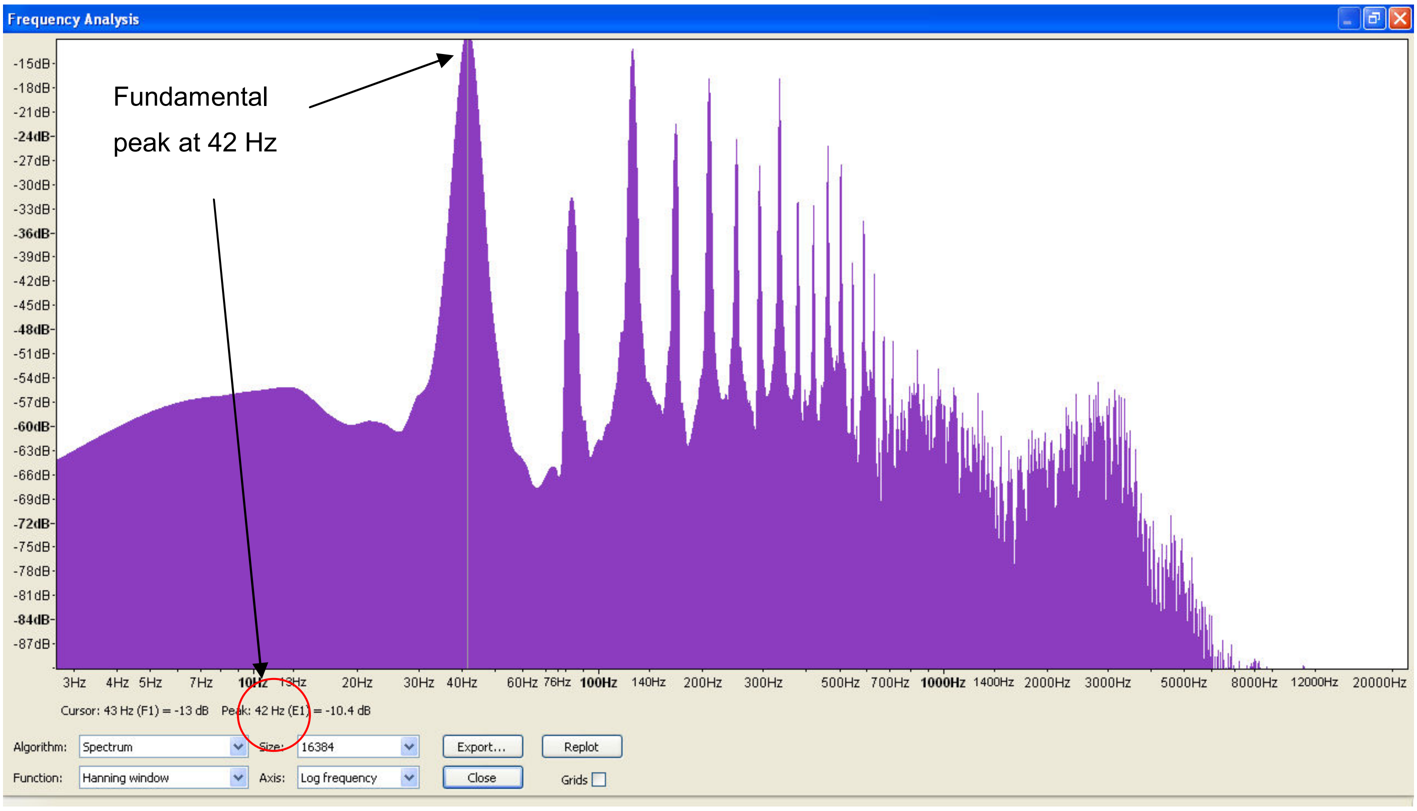
5.2 The lowest harmonic

The first question is why the lowest harmonic is so far from the prediction of the closed-open pipe? The answer is that the measured lowest harmonics are not a natural resonance of the instrument at all, but a heterodyne tone produced from higher resonances of the instrument called the *pedal note* (see Section 3.8).

One of the difficulties in proving that this is a pedal note is in producing the theoretical fundamental, as it is unplayable by the lips alone. Benade suggests that the fundamental can be played on a brass instrument using a clarinet mouthpiece [Benade, 1990]. This was attempted by placing a clarinet mouthpiece on the end of the horn mouthpipe:

Figure 38 - Clarinet mouthpiece

After a number of squeaks and groans the instrument produced a note at approximately 42 Hz as shown in Figure 39. The note can be heard on track 10 of the CD.



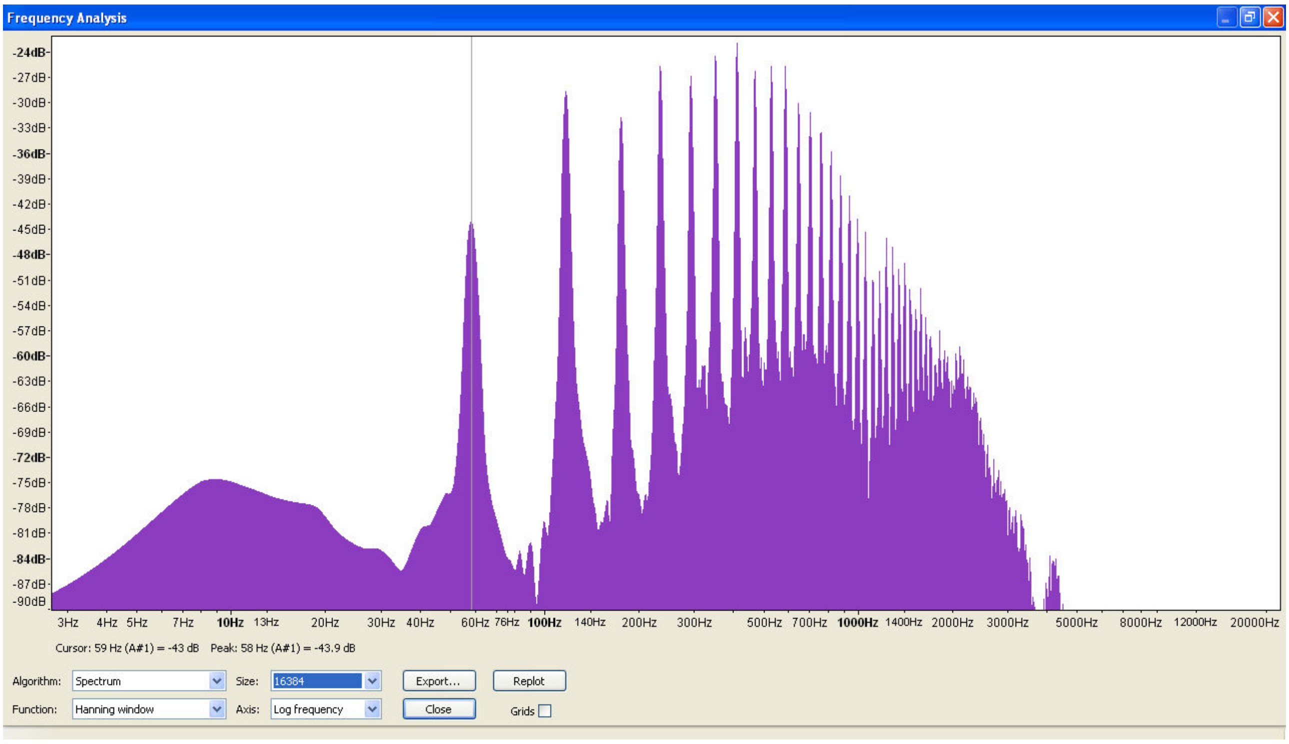
Fundamental peak at 42 Hz

Cursor: 43 Hz (F1) = -13 dB Peak: 42 Hz (E1) = -10.4 dB

Algorithm: Spectrum
Size: 16384
Function: Hanning window
Axis: Log frequency

Figure 39 - Proof of fundamental tone

Further proof of the pedal tone can be obtained by studying the FFT plot of the pedal note itself. This is shown in Figure 40.



Cursor: 59 Hz (A#1) = -43 dB Peak: 58 Hz (A#1) = -43.9 dB

Algorithm: Spectrum
Size: 16384
Function: Hanning window
Axis: Log frequency
Grids ☐

Figure 40 - FFT plot of pedal tone

In order for a strong heterodyne tone to be produced it would be necessary for a number of adjacent partials to be spaced by the pedal note frequency. Referring to the FFT plot, resonances 2 to 5 are at the following frequencies:

Table 5 - Difference frequencies between partials

Resonance no.	Frequency (Hz)	Difference frequency (Hz)
2	117	-
3	175	58
4	234	59
5	293	59
6	351	58

The results indicate that several difference frequencies are produced at 58 or 59 Hz.

5.3 Measuring the real fundamental

The measurements set out in Section 4 were repeated using the clarinet mouthpiece to find the missing fundamental. In all cases the fundamental could be played and can be heard on the CD as follows:

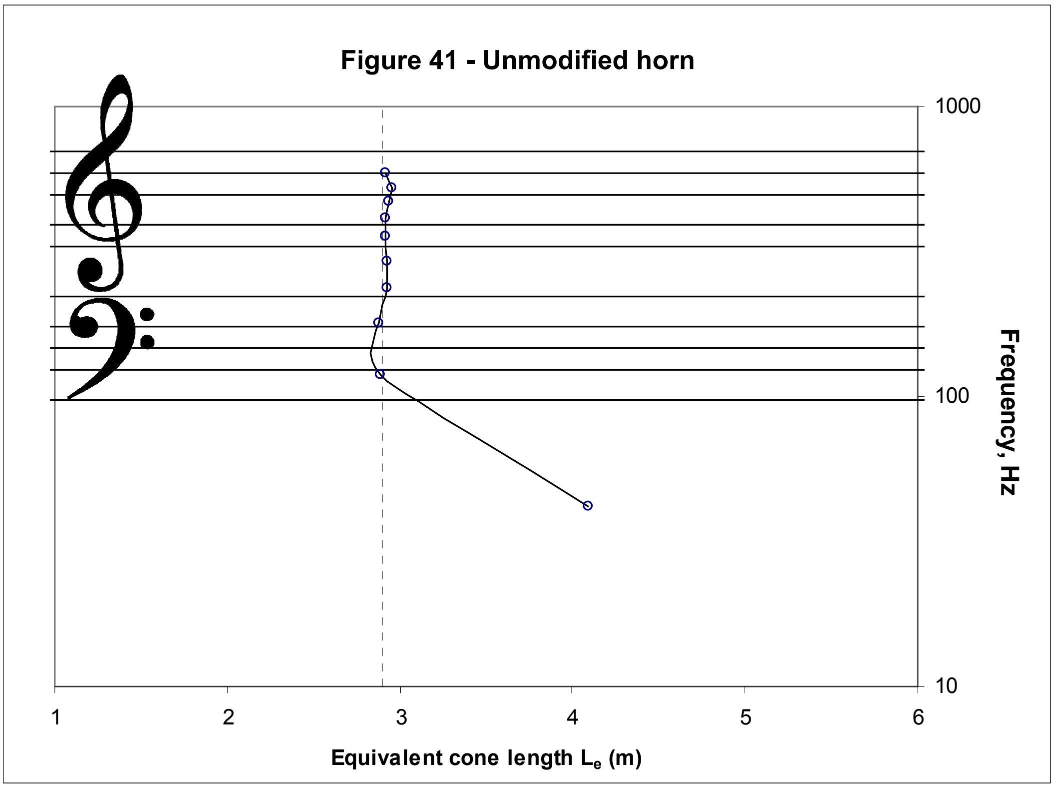
- Track 10 – unmodified horn
- Track 11 – hose mouthpipe / hose bell
- Track 12 – horn mouthpipe / hose bell

The revised measured harmonic series are shown below, along with the theoretical odd harmonic series obtainable from a closed-open pipe 2.7 m length:

Table 6 - Revised harmonic series to include ‘real’ fundamental

	Horn		**Hose mouthpipe Hose bell**		**Horn mouthpipe Hose bell**		**Hose mouthpipe Horn bell**		**Theoretical closed-open pipe**	
Harmonic no.	**F, Hz**	**Note**	**F, Hz**	**Note**	**F, Hz**	**Note**	**F, Hz**	**Note**	**F, Hz**	**Note**
1	42	E1	29	B♭0	37	D1	28	A0	32	C1
Pedal	*58*	*B♭1*	*51*	*A♭1*	*57*	*A1*	*47*	*G♭1*	*N/A*	*N/A*
2	119	B♭2	100	A♭2	115	B♭2	100	G2	96	G2
3	179	F3	158	E♭3	176	F3	159	E♭3	159	E♭3
4	235	B♭3	218	A3	236	B♭3	220	A3	223	A3
5	294	D4	288	D4	296	D4	288	D4	287	D4
6	354	F4	351	F4	354	F4	353	F4	350	F4
7	413	A♭4	413	A♭4	412	A♭4	416	A♭4	414	A♭4
8	469	B♭4	482	B4	475	B♭4	478	B♭4	478	B♭4
9	524	C5	544	D♭5	534	C5	535	C5	541	D♭5
10	590	D5	608	E♭5	621	E♭5	595	D5	605	E♭5

The following graphs show the equivalent cone length of the harmonic series of each 'instrument' including the fundamental and omitting the fictitious pedal note:



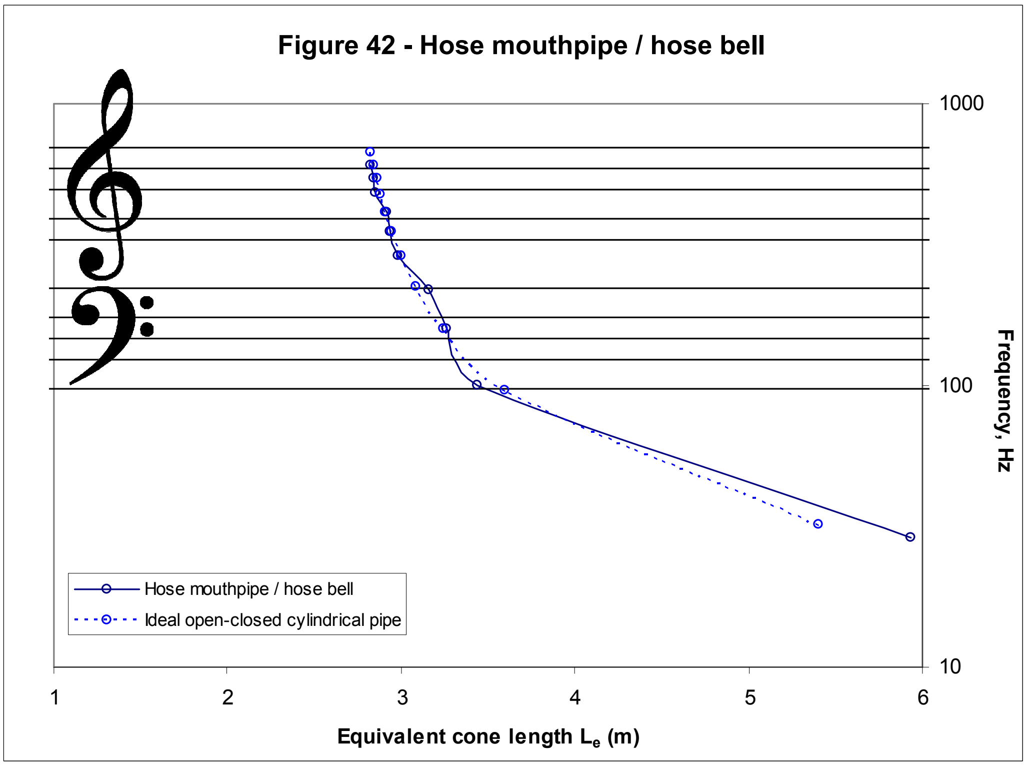


Figure 44 - Horn mouthpipe / hose bell

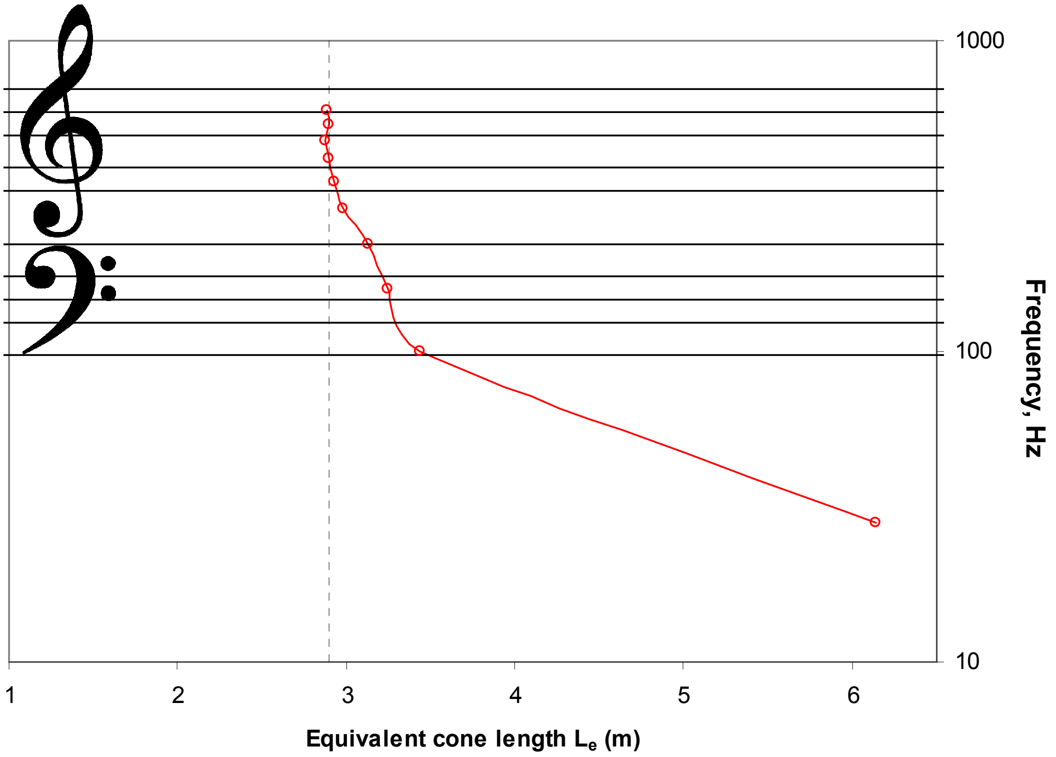
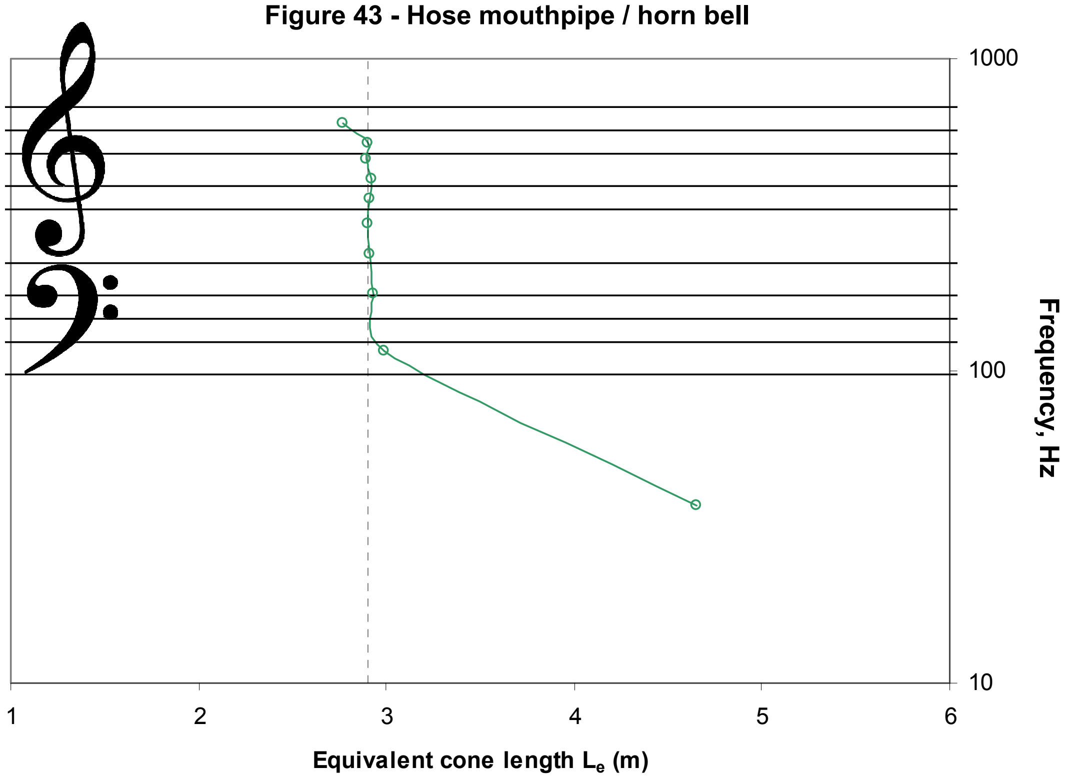
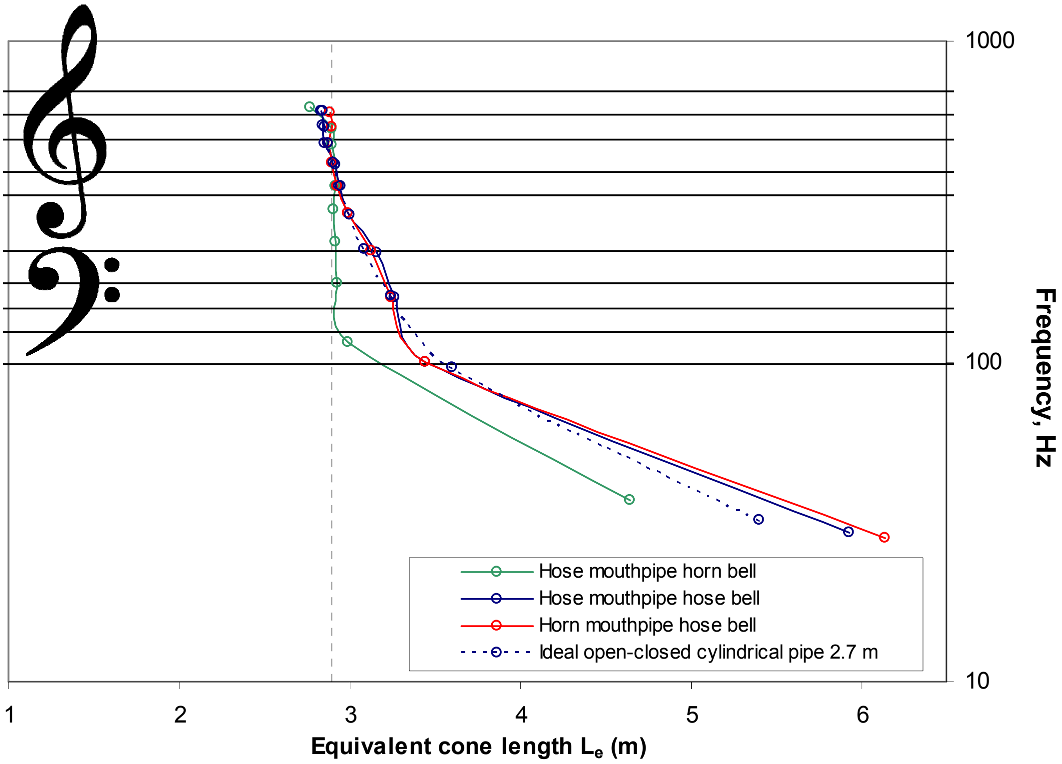


Figure 43 - Hose mouthpipe / horn bell



Re-comparing the three measurements in Section 5.1:

Figure 45 – Comparison of measurements

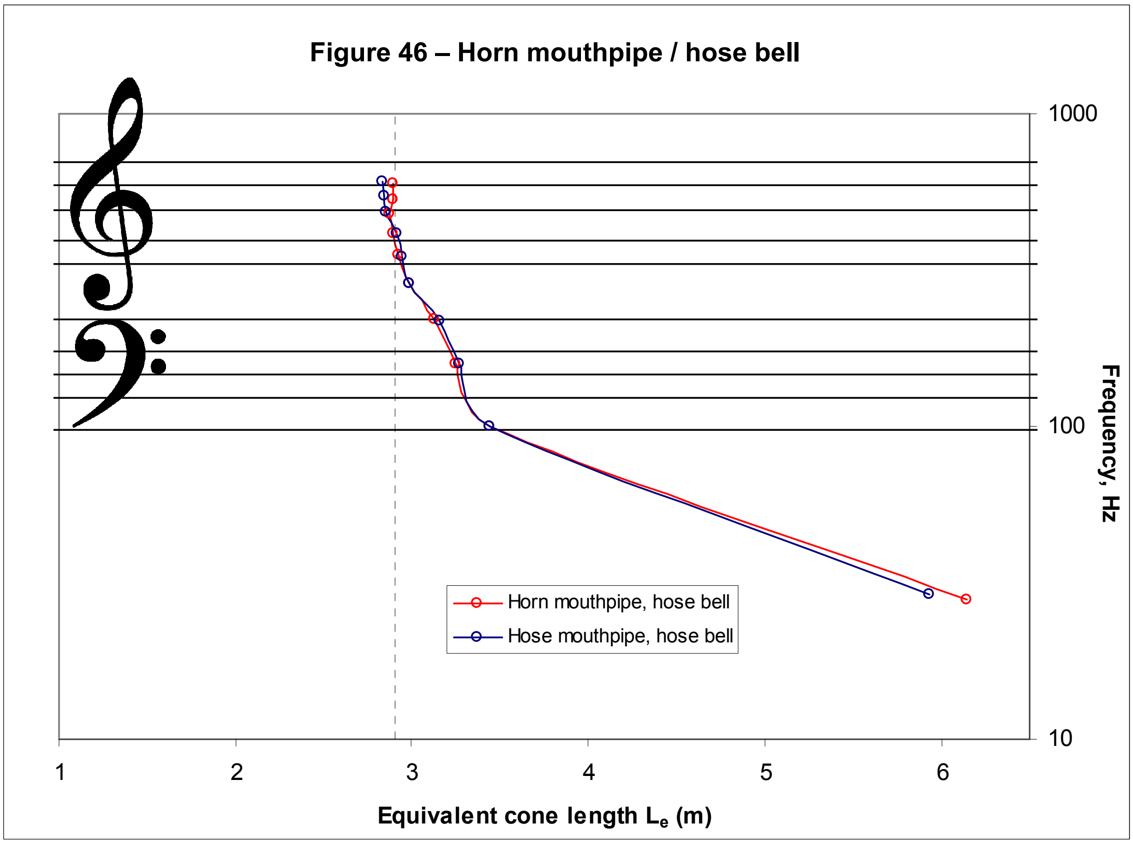


It can be seen that for the hose mouthpipe / hose bell and horn mouthpipe / hose bell, the lowest harmonic is now much closer to that predicted for the closed-open pipe, but there is still a difference. One reason for the differences may be that the clarinet mouthpiece causes an actual, and possibly an acoustic, change in length of the instrument, and hence the resonant frequency. Another experiment would have to be devised to investigate this further.

In the case of the hose mouthpipe / horn bell, the bell is raising the pitch of the fundamental in the same way as the higher resonances.

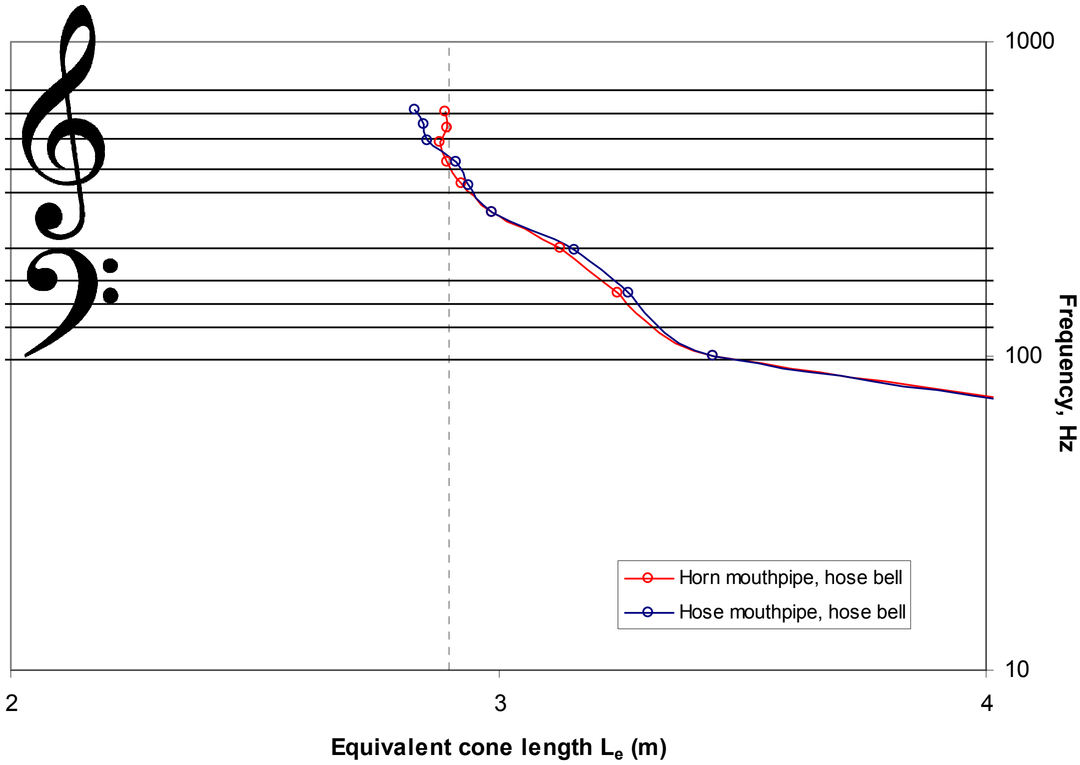
5.4 Effect of the horn mouthpipe

The horn mouthpipe appears to have very little influence on the lower harmonics, but a flattening effect on the upper harmonics. The equivalent cone lengths of the horn mouthpipe/hose bell and hose mouthpipe/hose bell are compared below:



The graph indicates that the mouthpipe makes little change to the harmonic series, except in harmonics 8-10, where they are slightly flattened (i.e. the equivalent cone length is lengthened). This can be observed by examining this range more closely:

Figure 47 - Horn mouthpipe / hose bell, 2 - 4 metres L_e



Many books state that the mouthpiece and mouthpipe significantly affect the intonation of brass instruments in the higher register. This does not appear to occur on the horn, as the higher harmonics can be played reasonably in tune by using the hosepipe mouthpipe / horn bell. However, Campbell and Greated state that one of the main differences between the horn and other brass instruments is the function of the mouthpiece and mouthpipe [Campbell/Greated 1987, p. 392].

The trumpet has a short tapering mouthpipe around two inches long, and this apparently plays an important role in correcting the intonation of the instrument. However, this is largely unnecessary in the horn as the bore profile can be designed to give an almost constant effective length in the upper register. The function of the horn mouthpipe is principally to provide an impedance multiplying effect in certain registers, and hence boost the natural resonances. Campbell / Greated state that the mouthpiece and mouthpipe have a strong impedance multiplying effect concentrated on the lowest three or four peaks. The impedance multiplication effect is in a higher register on the trumpet, where it adds to the brilliance of the sound. This is presumably one of the reasons why the trumpet has a much brighter tone than the horn.

The resonant frequency of the horn mouthpiece is therefore chosen to lie at the upper end of the playing range, and below this frequency the variation of effective length is relatively small. The popping frequency of the horn mouthpiece was observed to be around 700 Hz and can be heard on track 13 of the CD. A high C on the horn, which is generally the highest note in the standard repertoire, is around 693 Hz, and the statement therefore seems to hold true.

This impedance-multiplying effect was noted in playing the instrument, as with the horn mouthpipe / hose bell it was easier to sound notes than on the hose mouthpipe / hose bell. It was also noted that the pedal note was much harder to play on this instrument, which could indicate that the lower harmonic frequencies were more clearly defined, and if not locked into a consistent difference between adjacent harmonics, would not produce such a strong heterodyne tone. This would need further experimentation to verify.

5.5 Effect of the bell

The horn bell appears to raise the pitch of the lower harmonics, and turns the hosepipe instrument with odd harmonics into a real instrument with the full set of odd and even harmonics. Figure 48 compares the hosepipe mouthpipe / hosepipe bell instrument, the unmodified horn and the hosepipe mouthpipe / horn bell:

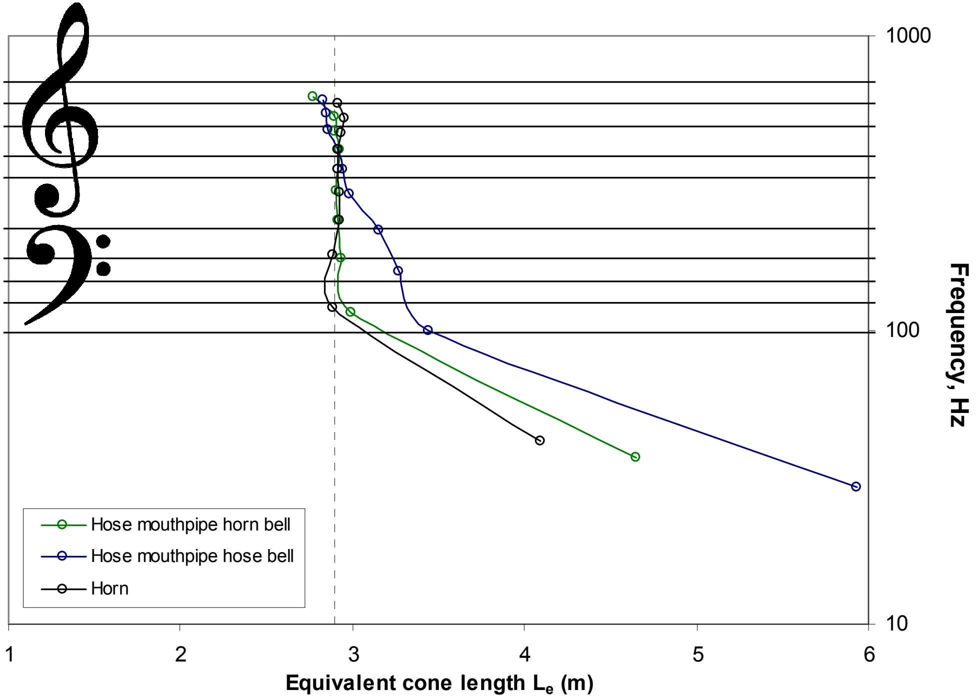


Figure 48 - Effect of the bell

The hose mouthpipe / horn bell harmonics become very close in pitch to those of the horn, and the effect can be heard clearly by the following auditory demonstrations:

In track 14, the hose mouthpipe / hose bell can be heard in the left channel, and the hose mouthpipe / horn bell in the right channel. This demonstrates that the intervals start far apart and become progressively closer together until they are very close by the 6th resonance.

In track 15, the hose mouthpipe / horn bell is heard in the left channel, and the unmodified horn is heard in the right channel. It can be heard that the resonances are very close together, indicating that the horn bell is pulling the pitch of the lowest resonances (below the 6th) up in frequency.

5.6 Sources of error

The main source of error in this experiment is in producing the resonances with the lip. Playing any instrument is not a mechanical process, and it is instinctive for the player to attempt to correct for intonation errors by adjusting the lip tension. It was noted that for the hosepipe instruments, for many harmonics the pitch was difficult to centre and easily variable, particularly in the higher register. The experiment could therefore be repeated with more accuracy by using the apparatus described in Section 3.4, or similar, to measure the input impedance of the instrument by purely mechanical and objective means.

5.7 Further work

The subject of instrument acoustics is well-researched, and there are many areas of possible further work to this study:

- Refining the measurements by using mechanical apparatus to measure the input impedance as described in Section 5.6. This would also enable a higher range of harmonics to be studied, as the experiment was limited by the notes playable with the lip.
- Repeating the study of the mouthpipe and bell effects for different brass instruments. The horn has a 'hybrid bore' (see Section 2.1.2), whereas instruments such as the trumpet have a much shorter mouthpipe and proportionally longer cylindrical section. Many reference works studied for this project indicate that the mouthpiece and mouthpipe have a significant effect on

the tuning of higher harmonics, but this study indicates that this does not happen on the horn.

- Repeating the study for the F side of the instrument. This would enable the effect of change of length in the central cylindrical section of pipe to be studied, since the mouthpipe and bell are the same for the F and B♭ sides.
- Predicting the behaviour of the instrument by mathematical solutions of the wave equation. The mathematical behaviour of waves in flaring horns is complex and it would be interesting to develop mathematical models to compare the theoretical and practical behaviour of the instrument.

6 CONCLUSIONS

- The study has compared the harmonic series of an open-closed pipe with that of a French horn.
- Analysis of the horn harmonics shows that it appears to produce a complete series of odd and even harmonics. This appears to contradict the theory of closed-open pipes which states that only odd harmonics can be produced.
- Further analysis showed that the lowest note playable on the B♭ horn is not a natural resonance of the instrument, but a heterodyne tone created by interaction between higher resonances. This note is known as the pedal note. The ‘real’ fundamental tone of the instrument is unplayable with the lip, but could be produced with a clarinet mouthpiece.
- A cylindrical closed-open pipe of similar length to the horn produces a comparable harmonic pattern, but the lower harmonics are flat and the higher harmonics are sharp.
- The tuning of the cylindrical closed-open pipe harmonics is altered by the bell and the mouthpipe. The bell raises the pitch of the first few harmonics, has little effect on the middle harmonics, and flattens the upper harmonics. The mouthpiece/mouthpipe flattens the upper harmonics but has little effect on the lower harmonics.
- The study therefore showed that a horn plays an approximately full odd and even harmonic series from the pedal note upwards.

APPENDIX A – REFERENCES / BIBLIOGRAPHY

Books / papers:

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APPENDIX B – INSTRUMENT MANUFACTURE

As part of this study, I interviewed the instrument builder Andy Taylor of Taylor Trumpets. Andy spent the first fifteen years of his career building French horns for Paxman in London, a well-respected horn brand. The purpose of the interview was to find out how the instrument maker approaches the design, which will have been determined over the centuries by tradition and experimentation, rather than application of acoustic theory. The following is a summary of our discussion, including some theories for the acoustic reasons for some of the issues raised.

Unlike the trumpet, the horn has seen little development over recent years and remains a very traditional instrument. There is therefore likely to have been little change in the instrument since the invention of the valve.

The instrument starts with the mouthpiece and mouthpipe. The mouthpiece consists of a cup, followed by a constricted throat, followed by a tapered section. The mouthpiece must be matched to the instrument, as the mouthpiece / mouthpipe combination is important for the intonation[2] of the instrument. Apparently this is less commonly a problem for the horn, which has evolved very little over the years, but an old mouthpiece in a new trumpet (or vice versa) can have an enormous effect on the playability of the instrument.

Note: this is observed in the measurements, in that the horn mouthpiece / mouthpipe had relatively little effect on the tuning of most of the harmonics. Presumably the mouthpiece / mouthpipe has more of an effect on the trumpet than the horn.

The mouthpiece fits into the mouthpipe which continues this taper and joins into the mainly cylindrical section of the instrument. This section is critical to make a good instrument – if not manufactured correctly the intonation of the instrument can be poor. This is perhaps the primary hallmark of a ‘bad’ instrument – one which is out of tune with itself will be difficult to play musically. Another feature of a poor instrument might be an uneven response in the playing – some notes might produce

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[2] This means that the instrument is not in tune with itself; i.e. the harmonics are out of tune relative to one another.

a sound of higher amplitude for a given effort than others (this is known on the violin as ‘wolf notes’).

Note: as discussed above, this mouthpipe effect appears to apply more to trumpets than French horns.

The next section of the instrument is the cylindrical section, which contains the valves and tuning slides. The valve section is often referred to by instrument builders as the ‘engine’ of the instrument. The position of the valves is important – at an early position in the pipe they will add resistance to the instrument. Larger bore instruments can be around 12 mm in diameter, older instruments tended to have a bore around 10.5 – 11 mm. Although this is a small change in cross sectional area, over a 12 foot pipe this represents a large change in volume. The bore width will not change the range of harmonics available, but a large bore instrument will tend to have a louder sound with a ‘darker’ tone, but will take more effort to play than a narrow bore instrument. Conversely, a narrow bore instrument will be easier to ‘leap’ between notes, and there is less air to move, meaning less effort in playing.

Any horn when played loudly will reach a point at which the tone will become very harsh. A smaller bore instrument will reach that point and ‘crack up’ at a lower volume than a wide bore instrument.

Note: although not specifically covered in the study, it is interesting to observe that this ‘breaking up’ or brassy tone occurs because of the very high sound pressure levels generated inside the instrument (Benade states that levels of 175 dB have been measured). At these levels the linear acoustics approximation breaks down and shock waves are formed. These non-linear shock waves cause the very brassy tone when the instruments are played loudly.

The ‘branch’ is where the instrument begins to flare from a cylindrical to a conical pipe, and eventually the rate of flare increases to the bell. The bell curve is approximately exponential, although the precise flare has been refined empirically over the years. Different parts of the bell affect different registers; at the point where the hand supports the bell, the mid range is affected, whereas low notes are affected by the open bell end. The bell is responsible for the tone quality of the instrument.

Note: this could be studied in further work to determine the bell’s effect on tone quality.

The horn is built from coils of pipe, and the bends have little effect on the playing qualities of the instrument. The coils must be braced at certain points, and the position of these braces can affect certain notes. For this reason, some trumpets have movable braces.

Note: presumably damping of the notes could occur if the braces are placed at pressure antinodes.

The material of the instrument can have an effect on the tone. A thicker material has a ‘darker’ sound and allows more volume before the tone ‘breaks up’. An instrument with thinner material becomes ‘raspier’ more easily, but has a brighter sound.

In trumpet manufacture the most common instrument is pitched in B♭. A less common pitch is trumpet in C. It is interesting to note that the C trumpet produced by shortening the cylindrical section of tubing to raise the pitch will suffer from poor intonation compared with the scaled down B♭ trumpet, where each of the component parts of the instrument are in proportion.

Note: this is likely to be due to the pitch-shifting effect of the mouthpiece/mouthpipe and bell. If a C trumpet is produced by shortening the cylindrical section only, the relative proportions of the mouthpiece/mouthpipe and bell will also be altered, and hence the tuning will be changed. It follows that an instrument where these sections are in proportion will work more satisfactorily.

APPENDIX C - MEASURING EQUIPMENT

Measurement location: AJA offices

Measurement date(s): 23 August 2010 / 9 September 2010

Measuring equipment used:

Equipment description / serial number	**Type number**	**Manufacturer**	**Date of calibration expiration**	**Calibration certificate number**
Laptop	6910p	Hewlett Packard	N/A	N/A
Sound card	UA25	Edirol	N/A	N/A
Microphone	MiniSPL	Neutrik	N/A	N/A
Audacity Digital Audio / FFT analysis software	N/A	N/A	N/A	N/A

Measurements taken by: Andy Thompson

APPENDIX D – CD CONTENTS

Musical excerpts

Track no.	Description	Credits	Points to listen for
1	**Handel** Allegro – Andante - Allegro from the Water Music	Schola Cantorum Basiliensis August Wenzinger	This would have been written for the natural horn. Notes produced are those in the natural harmonic series only (the open notes). See Section 2.1.2.
2	**W. A. Mozart** Horn Concerto no. 2, First Movement	Mozart – Horn Concertos & Concert Arias Timothy Brown / Orchestra of the Age of Enlightenment / Sigiswald	This is a concerto written for the hand horn player, so the open notes are modified with the hand to produce full musical scales. Listen out for muffled notes, where the hand nearly completely closes the bell to lower the pitch (for example, at time 5:03). See Section2.1.2.
3	**Richard Strauss** Horn Concerto no. 2, Third Movement	Strauss – Horn Concertos Herman Baumann	This track illustrates the versatility of the valved horn, which is capable of playing a full chromatic scale. See Section 2.1.3.
4	**Richard Strauss** Also Sprach Zarathustra (excerpt)	Also Sprach Zarathustra / Don Juan Berlin Philharmonic / Herbert Von Karajan	This track illustrates the full, brassy sound of the instrument when played loudly, capable of dominating the orchestra. See Section 2.5.
5	**Sibelius** Violin Concerto (excerpt)	Concerto in D minor for Violin and Orchestra Lahti Symphony Orchestra / Leonidas Kavakos / Osmo Vänskä	This track illustrates the quiet tone of the instrument, which in this section is accompanying the violin along with the bassoons. See Section 2.5.

Harmonic series

Track no.	Description	Points to listen for
6	Harmonic series – unmodified horn	The instrument plays a full set of odd and even harmonics from the pedal tone upwards. See Section 4.4.
7	Harmonic series – hose mouthpipe / hose bell	The instrument effectively behaves as a cylindrical pipe, and plays odd harmonics only. See Section 4.5.
8	Harmonic series – horn mouthpipe / hose bell	The instrument produces a similar set of harmonics as the hose mouthpipe / hose bell, but is easier to pitch. See Section 4.6.
9	Harmonic series – hose mouthpipe / horn bell	The lower harmonics are raised in pitch up to about the 6th harmonic, after which they follow those of a cylindrical pipe. See Section 4.7.
10	Fundamental played with clarinet mouthpiece: horn	See Section 5.3.
11	Fundamental played with clarinet mouthpiece: hose mouthpipe / hose bell	See Section 5.3.
12	Fundamental played with clarinet mouthpiece: horn mouthpipe / hose bell	See Section 5.3.
13	Mouthpiece popping frequency	Produced by slapping the mouthpiece against the hand. The mouthpiece forms a Helmholtz resonator. The natural frequency is called the ‘pop tone’. See Section 3.11.
14	Left channel: hose mouthpipe / hose bell harmonic series Right channel: hose mouthpipe / horn bell harmonic series	Note how the harmonics start far apart at low frequencies but become closer together by about the 6th resonance. This is due to the sharpening effect of the bell on the low resonances. See Section 3.14.
15	Left channel: unmodified horn harmonic series Right channel: hose mouthpipe / horn bell harmonic series	Note how the harmonic series are similar, again illustrating that the bell modifies the harmonic series to near that of an ideal cone. See Section 3.14.

APPENDIX E – GLOSSARY OF TERMS

The following is a short glossary of some musical and technical terms used in this project.

Chromatic:	In music, the *chromatic scale* is a sequence of notes where each subsequent note is a semitone apart.
Concert pitch:	The pitch of a note as played on a piano.
Flat:	Lower in pitch.
Fundamental:	The lowest resonance of a pipe.
Harmonic:	In the context of this report, a harmonic is a natural resonance of a pipe. The first harmonic is the *fundamental.* Musicians often refer to the natural notes of an instrument as harmonics.
Heterodyne:	In a nonlinear system, two waves of different frequency combine to produce new frequencies at the sum and difference between the two.
Intonation:	A judgement of the relative tuning of notes in an instrument. For example, the tuning of an instrument may be judged to have ‘good’ intonation if the frequencies of notes are exact multiples of one another.
Mode:	See *harmonics*.
Open notes:	See *harmonics*.
Overtone:	Resonances higher than the fundamental.
Partial:	When a pipe resonates in several *modes*, each mode is a *partial*, or ‘partial vibration’.
Register:	A particular frequency range of an instrument (e.g. high / low register).
Semitone:	The smallest musical interval between two notes in Western tonal music. One octave comprises twelve semitones. In frequency terms, two frequencies are a semitone apart when their relative ratio is 1.0595 (for equal temperament).
Sharp:	Higher in pitch.
Tone:	One tone = two semitones.